

# A Pseudo-Dynamic Method to Analyze Retaining Wall with Reinforced and Unreinforced Backfill

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**ABSTRACT:** *In this article, the problem of determining pseudo-dynamic pressure and its associated forces on a rigid vertical retaining wall is solved analytically using the horizontal slices method for both reinforced and unreinforced walls. The use of this method in conjunction with the suggested equations and unknowns offers a pseudo-dynamic method that is then compared with the results of an available software. In the proposed method, different seismic accelerations have been modeled at different soil structure heights. Reinforced soil pressure on a retaining wall and the angle of the critical failure wedge are calculated using the new formulation. It is shown that as the horizontal seismic acceleration coefficient increases the angle of the critical failure wedge is reduced and that the maximum extension force can be increased for each layer by using stronger and longer reinforcements. The results of the pseudo-dynamic method show that both vertical and horizontal seismic accelerations are essential coefficients for calculation of the required length and extension force of the reinforcements and that their importance increases as the vertical and horizontal seismic accelerations increase. Also, the location of the application point of the resultant pressure rises as the horizontal seismic acceleration coefficient increases.*

**Keywords:** Active earth pressure; Reinforced soil; Parallel slices; Retaining wall

## 1. Introduction

For many decades the seismic analysis of retaining walls has been based on the simple extension of Coulomb's limit-equilibrium analysis, also known as the Mononobe and Matsuo [1] and Okabe [2] procedures. Recent research by Richards et al [3], Choudhury and Singh [4], and others also mention the pseudo-static procedure to estimate seismic active earth pressure behind a retaining wall. However, in this method, the dynamic behavior of seismic loading has been considered in an approximate manner without considering the possible effect of time [5]. Some solutions based on the pseudo-static method have also been presented for retaining walls with reinforced backfill, but the effect of dynamic behavior has not been considered. In this paper, by modifying the unknowns and equations of horizontal slices method introduced by Shahgholi et al [6], Nouri et al

[7], and Shekarian et al [8], a new formula for calculating the seismic active earth pressure behind rigid retaining walls and retaining walls with reinforced backfill is presented.

## 2. Previous Research

The horizontal slices method was suggested by Lo and Xu [9]. Shahgholi et al [6] introduced a new analytical method based on the limit equilibrium approach that evaluated the seismic stability of reinforced soil walls. On the basis of this method, equations and unknowns in the simplified formulation were introduced using a known value for  $\sum T_i$  to determine the values of unknowns  $N_i$ ,  $S_i$ , and  $F_s$ .  $\sum T_i$  is the sum of the extension forces on one assumed failure wedge. Table (1) shows the equations and unknowns of the simplified formulation.

**Table 1.** Equations and unknowns of simplified formulation of the horizontal slices method by Shahgholi et al [6].

Unknowns	No.	Equations	No.
$N_i$ Normal Forces Upon Base of Each Slice	$n$	$\sum F_y = 0$ For Each Slice	$n$
$S_i$ Shear Forces Upon Base of Each Slice	$n$	$\tau_m = \tau_f / F_s$ For Each Slice	$n$
$F_s$ Safety Factor	1	$\sum F_x = 0$ For Each Slice	1

The equations and unknowns for a complete formulation of the horizontal slices method were presented by Nouri et al [7]. In this research, two formulations using  $5n-1$  and  $3n$  to calculate  $(\sum T_i)_{max}$  were assessed, where  $F_s$  on the critical failure wedge is unity (1). In the  $3n$  formulation, the unknowns are  $N_i$ ,  $S_i$  and  $H_i$ , where  $H_i$  is the inter-slice shear force and the known is  $\sum T_i$ . In the  $5n-1$  formulation, in addition to the above parameters,  $V_i$ ,  $X_{V_i}$  and  $\lambda$  are unknowns.  $V_i$  is the vertical inter-slice force;  $X_{V_i}$  is the coordinate of the point where  $V_i$  acts on the base of the slice; and  $\lambda$  is the Morgenstern and Price factor.

Saran et al [10] investigated the rate of reduction pressure on the wall caused by reinforcement. Their solution to the design of walls in front of cohesionless soil ( $c=0$ ) assumes homogeneity and a planar failure wedge. Also, by disregarding second-order and higher order terms, the static equilibrium of a small element of failure wedge is yielded. The earth pressure due to the surcharge will be reached by integrating the earth pressure equation over the wall height. These studies show that the overturning moment in reinforced soil is less than unreinforced backfill. The application point of imposed pressure in this method is higher than one-third the height of the wall.

Garg [11] investigated an 11m rigid retaining wall. In this investigation, the soil behind the wall was assumed to be homogenous, isotropic and cohesionless and the failure wedge was assumed to be planar. By adjusting the non-dimensional curves presented by this researcher, the resultant pressure and point of application can be obtained. These results were similar to those provided by Saran et al [10].

Mononobe-Okabe (M-O) [1-2] were early pioneers in calculating seismic active earth pressure on rigid retaining walls to obtain an active earth pressure coefficient under seismic conditions. An extension of Coulomb's method was used for the static case to determine the earth pressure by assuming the equilibrium of a triangular failure wedge.

Nimbalkar et al [12] investigated time effect and phase change in shear and primary waves propagating in the backfill behind a rigid retaining wall and the effects of both horizontal and vertical seismic coefficients were studied. Dynamic active earth pressure on retaining structures was studied by Choudhury and Chatterjee [13]. The non-dimensional design chart proposed by that research can be used to estimate the whole dynamic earth pressure acting on the retaining wall. Mylonakis et al [14] also proposed a limit analysis solution for determining gravitational and earthquake-induced earth pressure on gravity walls retaining cohesionless soil.

### 3. Principle Equations and Assumptions Determining Earth Pressure Using the Horizontal Slices Method

Two formulations of the horizontal slices method were considered to obtain lateral earth pressure on a wall in two cases. The following assumptions were made:

- 1) Vertical stress on each slice is assumed to be  $\gamma z$  (for the vertical wall equal to zero).
- 2) The failure surface is assumed to be planar.
- 3) The safety factor is assumed to be equal for all slices.
- 4) The method is limited to homogenous masses.
- 5) Analysis is done on the basis of limit equilibrium.
- 6) The failure surface is assumed to pass through the base of the wall.
- 7) The cohesion of the backfill is assumed to be equal to zero.
- 8) The horizontal inter-slice force is disregarded in equations and unknowns ( $H_i = H_{i+1}$ ).

### 4. Analytical Method for Rigid Retaining Wall with Reinforced Backfill

In these types of walls, if the base is subjected to harmonic horizontal and vertical seismic accelerations of amplitudes  $\alpha_h$  and  $\alpha_v$ , the accelerations at depth  $z$  below the top of the wall can be explained as [12]:

$$\alpha_h(z, t) = \alpha_h \sin \omega(t - \frac{H-z}{V_s}) \quad \alpha_h = K_h g$$

$$\alpha_v(z, t) = \alpha_v \sin \omega(t - \frac{H-z}{V_p}) \quad \alpha_v = K_v g$$

where  $V_s$  and  $V_p$  are the shear wave and primary wave velocity, respectively. For most geotechnical materials, it is assumed that  $\frac{V_p}{V_s} = 1.87$  [15]. It is also assumed that  $\omega$  is the angular frequency of base shaking equal to  $\frac{2\pi}{T}$  where  $T$  is the period of lateral shaking. For most geotechnical structures  $T=0.3s$  is

a reasonable value [12, 16]. Note that  $t$  in this formula is assumed to be one second.

For the  $i^{th}$  slice, the mass of the elemental is:

$$m_i(z) = \frac{\gamma}{g} \frac{H-z}{\tan\alpha_i} h_i$$

Thus, the total force  $q_{hi}$  acting on the  $i^{th}$  slice can be explained as:

$$q_{hi} = m_i(z) \cdot \alpha_h(z, t)$$

The total force  $q_{vi}$  acting on the reinforced soil wall can be explained as:

$$q_{vi} = m_i(z) \cdot \alpha_v(z, t)$$

Forces equilibrium equations in the horizontal and vertical directions and the moment equilibrium for  $i^{th}$  slice become:

$$\sum F_x = 0 \Rightarrow F_i \cos\delta + T_i + S_i \cos\theta - N_i \sin\theta - q_{hi} = 0$$

$$\sum F_y = 0 \Rightarrow F_i \sin\delta + V_{i+1} + S_i \sin\theta + N_i \cos\theta - V_i - W_i - q_{vi} = 0$$

$$\sum M_o = 0 \Rightarrow V_{i+1} \cdot X_{V_{i+1}} - V_i \cdot X_{V_i} - (W_i + q_{vi}) \cdot X_{G_o} + \left( \frac{N_i}{\sin\theta} - F_i \cos\delta - T_i \right) \left( \sum_1^{i-1} h_i + \frac{h_i}{2} \right) + q_{hi} \left( \sum_1^{i-1} h_i + \frac{X_{V_{i+1}} h_i^2 + \frac{2}{3} (X_{V_i} - X_{V_{i+1}}) h_i^2}{2 X_{V_{i+1}} h_i + (X_{V_i} - X_{V_{i+1}}) h_i} \right) = 0$$

In these relations,  $S_i$  is the shear force on the base of the  $i^{th}$  slice and should be equal to  $\frac{N_i \tan\phi}{F_s}$ . Figure (1) depicts all defined parameters.  $X_{V_i}$  and  $X_{G_o}$  are evaluated as:

$$X_{V_i} = \frac{\sum_1^i h_i}{2 \tan\theta}$$

$$X_{G_o} = \frac{\sum_1^{i-1} h_i + \frac{h_i}{2}}{2 \tan\theta}$$

The angle of failure wedge ( $\theta$ ) is described in relation to  $\sum T_i$  on the critical failure wedge and the measurement of  $\sum F_i$ . The critical  $\theta$  is solved as:

$$\frac{\partial(\sum(T_i + F_i))}{\partial\theta} = 0$$

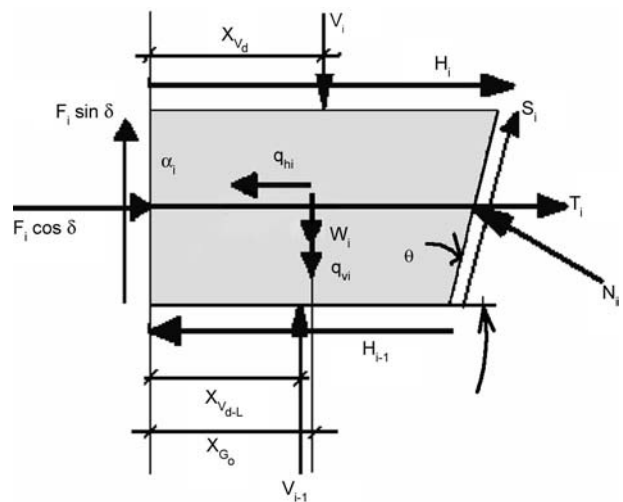


Figure 1. Forces acting on a single horizontal slice with reinforcement.

$F_i$  for the  $i^{th}$  slice can be obtained by solving  $4n$  equations for  $4n$  unknowns. Table (2) presents the equations and unknowns of the complete formulation.

## 5. Analytical Method for a Rigid Retaining Wall with Unreinforced Backfill

As in the previous case, if the base is subjected to harmonic horizontal and vertical seismic accelerations of amplitudes  $\alpha_h$  and  $\alpha_v$ , the accelerations at depth  $z$  below the top of the wall can be explained as proposed method and [12]:

$$\alpha_h(z, t) = \alpha_h \left( 1 + \beta \frac{H-z}{H} \right) \sin\omega \left( t - \frac{H-z}{V_s} \right) \quad \alpha_h = K_h g$$

$$\alpha_v(z, t) = \alpha_v \left( 1 + \beta \frac{H-z}{H} \right) \sin\omega \left( t - \frac{H-z}{V_p} \right) \quad \alpha_v = K_v g$$

All parameters are similar to those for retaining walls with reinforced backfill, except that for  $\alpha_h$  and  $\alpha_v$ ,  $\beta$  is assumed to be 1.4.

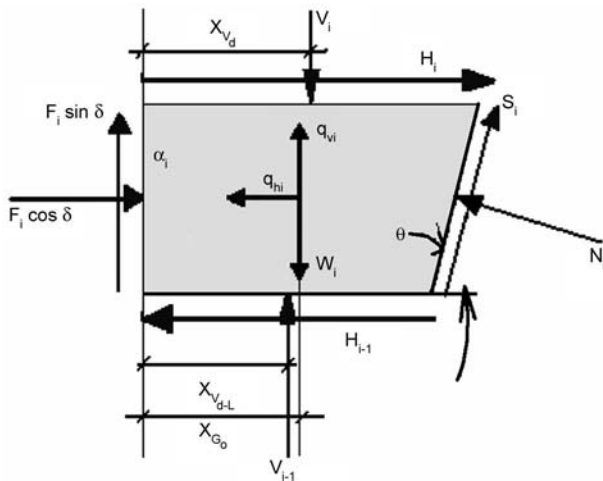
Table 2. Equations and unknowns of complete formulation of parallel-to-slope method to reach horizontal pressure on wall.

Equations	Number
For Each Slice: $\sum F_x = 0$	$n$
For Each Slice: $\sum F_y = 0$	$n$
For Each Slice: $\sum M_o = 0$	$n$
For Each Slice: $\tau_m = \frac{\tau_f}{F_s}$	$n$
Unknowns	Number
Reinforcements Force ( $T_i$ )	$n$
Normal Forces Upon Base of Each Slice ( $N_i$ )	$n$
Shear Forces Upon Base of Each Slice ( $S_i$ )	$n$
Pure Force on Wall ( $F_i$ )	$n$

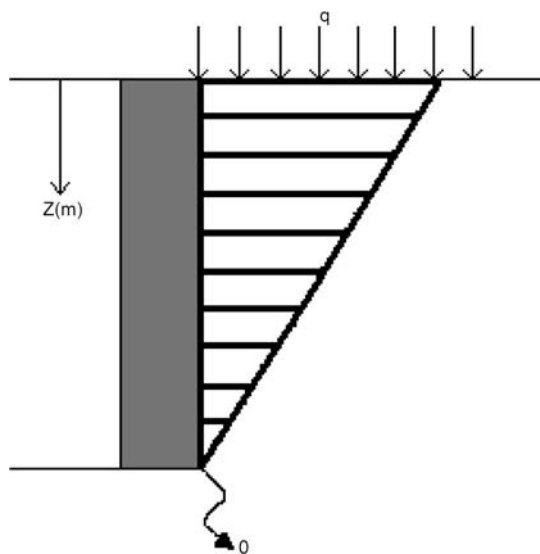
In this section,  $3n$  unknowns must be solved using  $3n$  equations. Table (3) shows the equations and unknowns of the complete formulation. Figures (2) and (3) show the face of the wall and details of the forces acting on a single slice.

**Table 3.** Equations and unknowns of complete formulation of horizontal slices method to reach resultant pressure on wall.

Equations	Number
For Each Slice: $\sum F_y = 0$	$n$
For Each Slice: $\sum M_o = 0$	$n$
For Each Slice: $\tau_m = \frac{\tau_f}{F_s}$	$n$
Unknowns	
Normal Forces Upon Base of Each Slice ( $N_i$ )	$n$
Shear Forces Upon Base of Each Slice ( $S_i$ )	$n$
Resultant Pressure Due to Each Slice on Wall ( $F_i$ )	$n$



**Figure 2.** Forces acting on a single horizontal slice.



**Figure 3.** Face of the wall.

Force equilibrium equations in the vertical direction and the moment equilibrium for the  $i^{th}$  slice become:

$$\sum F_y = 0 \Rightarrow F_i \sin \delta + V_{i+1} + S_i \sin \theta - N_i \cos \theta - V_i - W_i - q_{vi} = 0$$

$$\sum M_o = 0 \Rightarrow V_{i+1} \cdot X_{V_{i+1}} - V_i \cdot X_{V_i} - (W_i + q_{vi}) \cdot X_{G_o} + \left( \frac{N_i}{\sin \theta} - F_i \cos \delta - q_{hi} \right) \cdot \left( \sum_1^{i-1} h_i + \frac{h_i}{2} \right) = 0$$

### 6. Comparison of Analytical Results with an Available Method for Retaining Wall with Reinforced Backfill

The results of the proposed pseudo-dynamic method were compared with the results of *MSEW* [17]. The details of the reinforced soil wall used in the analysis to verify the extension forces on the critical failure wedge and to identify this critical failure wedge are presented in Table (4).

In Table (5), the summation of all  $T_i$  on the critical failure wedge for two methods at different seismic accelerations (without surcharge) is evaluated. As shown in both, an excellent accommodation for static cases (when  $K_h$  is equal to zero) is produced. But for  $K_h \neq 0$ , the difference between the two methods increases and, in all cases, the pseudo-dynamic method presents higher values of  $\sum T_i$  than does the *MSEW* program.

The differences between these two methods are 5% to 16% for reinforced soil retaining wall without surcharge. Figure (4) diagrams the critical failure wedge variations for the angle of internal friction and compares it to the *MSEW* program. When  $K_h = 0.15$ , the proposed method presents lower values than the *MSEW* [17] and Rankine methods ( $45 + \frac{\phi}{2}$ ). Figure (5) (for the static case) verifies the variation of active earth pressure on wall and active earth pressure of the reinforced soil ( $K_r$ ) reached by two methods (proposed method and [18]).

### 7. Comparison of Methods for Retaining Walls with Unreinforced Backfill

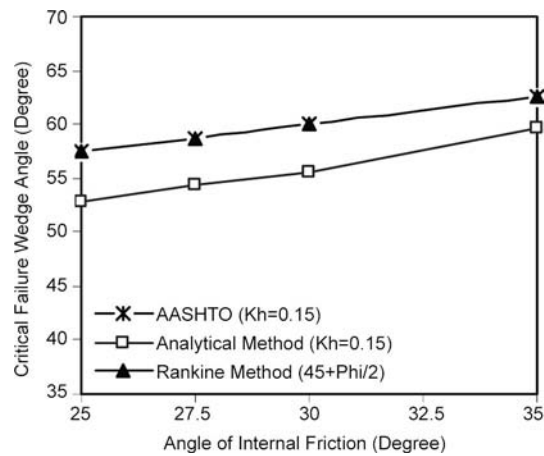
Table (6) presents the results for active earth pressure ( $K_a$ ) for the proposed method and other methods. The results of proposed method are in good agreement with the zero extension line solution of Habibaghahi and Ghahramani [19]. The maximum difference with other methods is about 10%.

**Table 4.** Comparison of  $\theta_{critical}$  reached using proposed and MSEW methods for walls without surcharge (Degrees).

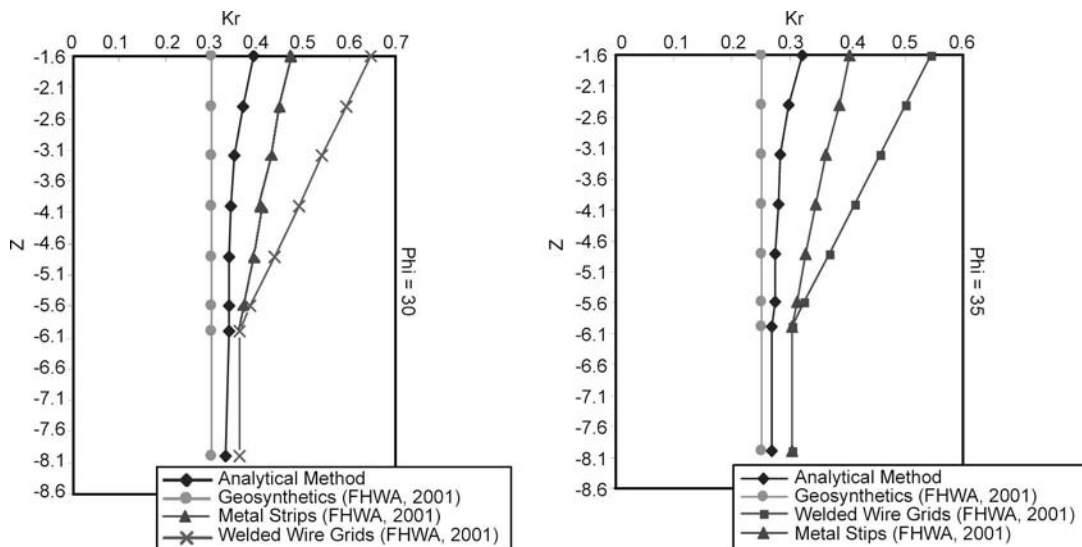
$\phi$	$\phi = 25$		$\phi = 27.5$		$\phi = 30$		$\phi = 35$	
Methods	Analytical	Software	Analytical	Software	Analytical	Software	Analytical	Software
$K_h = 0$	57.87	55.23	57.01	56.35	60.16	57.5	62.45	60.75
$K_h = 0.05$	54.43	54.46	56.14	56.34	58.44	57.91	60.16	61.18
$K_h = 0.15$	52.71	54.46	54.43	56.34	55.57	57.91	59.58	61.18
$K_h = 0.25$	50.42	54.46	51.56	56.34	52.14	57.91	53.85	61.18

**Table 5.** Comparison of  $\sum T_i$  for proposed and MSEW methods for walls without surcharge (KN/m).

$\phi$	$\phi = 25$		$\phi = 27.5$		$\phi = 30$		$\phi = 35$	
Methods	Analytical	Software	Analytical	Software	Analytical	Software	Analytical	Software
$K_h = 0$	179.02	179.81	162.77	163.78	146.16	148.85	116.71	122.00
$K_h = 0.05$	192.20	204.20	170.53	186.70	156.71	170.45	126.14	141.14
$K_h = 0.15$	218.11	247.75	198.86	227.69	180.74	209.3	148.82	175.32
$K_h = 0.25$	249.22	284.34	228.16	262.10	208.40	241.42	171.56	204.25



**Figure 4.** Variations of critical failure wedge angle in relation to angle of internal friction for AASHTO [20], Rankine and proposed methods.

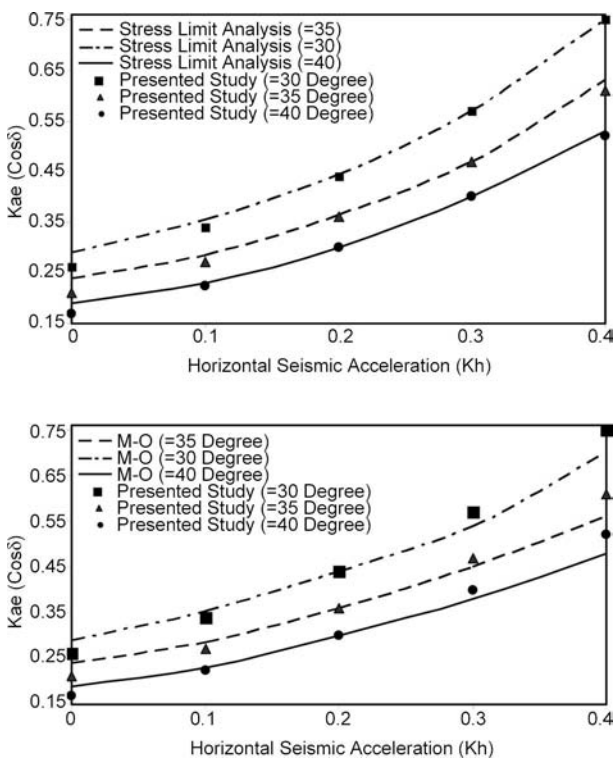


**Figure 5.** Comparison of active earth pressure coefficient in depth for proposed and FHWA [18] methods for different reinforcements.

The results for seismic active earth pressure versus horizontal seismic acceleration coefficient are presented in Figure (6). Naturally, active pressure increases with increasing levels of seismic acceleration and reduces with an increasing friction angle.

**Table 6.** Comparison of results for active pressures predicted by various methods.

$Kh = Kv = 0$	$\phi = 20^\circ$		$\phi = 30^\circ$		$\phi = 40^\circ$	
	$\delta = 0$	$\delta = 10$	$\delta = 0$	$\delta = 15$	$\delta = 0$	$\delta = 20$
Habibagahi and Ghahramani [19]	0.49	0.41	0.33	0.27	0.22	0.17
Mylonakis et al [14]	0.49	0.45	0.33	0.30	0.22	0.20
Proposed Method	0.50	0.40	0.33	0.27	0.22	0.18



**Figure 6.** Comparison of seismic active earth pressure coefficient predicted by stress limit analysis, Mononobe-Okabe (M-O) [1-2] and proposed methods for different horizontal seismic acceleration coefficients ( $\delta = \phi/2$ ).

### 8. Conclusions

This paper proposes a simple method that includes a pseudo-dynamic approach considering time effect and phase change in shear and primary waves propagating in the reinforced or unreinforced backfill behind a rigid retaining wall. To determine the extension force of reinforcements in a retaining wall with reinforced backfill, a new procedure based on the horizontal slices method described by Nouri et al

[7] and Shahgholi et al [6] is introduced.

A pseudo-dynamic method was suggested and compared with the results of *MSEW*. A negligible difference was observed between the two methods under static conditions. In the *MSEW* program, the horizontal seismic acceleration coefficient should be constant for the whole failure wedge because of the use of the vertical slices method. In the new method, the use of the horizontal slices method allows for the modeling of different seismic accelerations at different heights of the soil structure. For a retaining wall with unreinforced backfill, a new formulation has been suggested. The results illustrate that active earth pressure varies with the changing friction angle and levels of seismic acceleration.

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