AN EQUIVALENT TRUSS MODEL FOR NONLINEAR STATIC ANALYSIS OF URM BUILDINGS

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ABSTRACT

According to the importance of seismic evaluation of existing unreinforced masonry (URM) buildings, researchers are mostly interested in numerical modelling of these types of structures and their components. On the other hand in seismic evaluation and retrofitting codes which are mostly based on Performance Based Seismic Design (PBSD), different analysis methods such as linear and nonlinear, static and dynamic analyses are recommended. Therefore, simple equivalent frame models with lower computational cost are very useful for modelling and analysis of unreinforced masonry buildings. In this article at first, a simple equivalent truss model is proposed for modelling of an unreinforced masonry wall with sliding shear failure as the dominant in-plane failure mode. Then, the proposed model is applied for push over analysis of an unreinforced masonry wall consists of some piers and the push over curve of the wall is determined and discussed.

INTRODUCTION

Considerable unreinforced masonry buildings with different occupancies such as residential or official exist in all over the world. Some of these buildings are historical monuments like churches, mosques and masonry towers. Unreinforced masonry walls are the most important load resisting elements in these buildings because they resist lateral loads due to earthquakes as well as the gravity loads from the floors. Therefore, considerable experimental researches are conducted to investigate the behaviour of unreinforced masonry walls especially against seismic loads.

Numerical modelling approaches are also developed for analysis of unreinforced masonry walls and buildings by application of different numerical tools such as Finite Element or Discrete Element methods. Modelling approaches that are recommended for unreinforced masonry walls can be categorized into three groups; micro and simplified micro modelling, macro modelling and equivalent frame modelling. In micro modelling approach the masonry unit, mortar joint and the unit-mortar bond are modelled separately with appropriate constitutive laws for each phase. In simplified micro modelling approach, each masonry unit and half of the mortar joints around it are considered as equivalent blocks that appropriate interface models join them together. Although the results of micro modelling approaches show high accuracy, but because of their complexity and much computational cost, they are not suitable for analysis of masonry buildings with several walls and stories. Thus macro modelling approach with less computational cost has been introduced. In this method masonry is assumed as a homogenised isotropic or orthotropic material and the equivalent mechanical properties for masonry assemblage are used in analyzes. The results obtained with this method have less precision than the results of micro modelling approaches. The last modelling method is called equivalent frame method. In this method, one dimensional elements such as frame or truss elements are utilized for modelling of URM Buildings. Because of its simplicity and low computational cost, this method is very attractive for nonlinear static and dynamic analysis of unreinforced masonry buildings. Therefore, considerable investigations are carried out for development of simple equivalent frame or truss models for analysis of URM buildings. Roca et al (2005) presented an equivalent frame model for nonlinear analysis of URM buildings and historical monuments. They developed their model in three following stages; derivation of appropriate relationships for one dimensional curved elements, simulation of the wall behaviour with one dimensional elements and development of appropriate constitutive law based on different failure modes of URM walls. Finally, a building was analysed with their proposed modelling approach and acceptable results were obtained. Also, Roca (2006) developed simple models for prediction of the ultimate capacity of URM walls based on simple equilibrium relationships. Chen et al (2008) introduced an equivalent frame model with appropriate plastic hinges for nonlinear analysis of URM buildings. Also, a simple method based on Mexican building code (SMSA) has been introduced by Tena-Colunga and Cano-Licona (2010). They modified the existing model by a modification factor applied to shear areas of the walls. It is noticeable that the existing model, only considered shear deformation of URM walls. Recently, Lagomarsino et al (2013) presented a model in software Tremuri for nonlinear analysis of URM buildings to evaluate their seismic performance.

Since most of the existing unreinforced masonry buildings showed vulnerable in earthquakes, seismic evaluation of these buildings and development of appropriate retrofitting methods for them are considered by many researchers and engineers. Therefore, seismic evaluation and retrofitting codes such as FEMA356, which are mostly based on Performance Based Seismic Design are introduced. In these instructions, different analysis methods such as linear and nonlinear, static and dynamic analyses are recommended for seismic evaluation of existing buildings. Therefore, simple equivalent frame models with lower computational cost with compare to the other complex modelling approaches are very useful for modelling and analysis of unreinforced masonry buildings.

In this paper, a simple equivalent truss model for nonlinear analysis of URM walls with sliding shear failure as the dominant in-plane failure mode is presented. At first stage, the model is developed for a single URM shear wall. Then combination of the walls in plan and elevation of a building with several walls is explained. The advantage of the current method with compare to the other equivalent frame methods is that in this method, only bar elements with axial plastic hinges are required for modeling of an URM wall. Also, the effect of gravity loads on the in-plane capacity of the masonry piers is considered automatically in the model.

IN-PLANE BEAHAVIOUR OF URM WALLS

In order to perform nonlinear analysis of URM buildings, elastic and inelastic behavior of URM shear walls as the main lateral load resisting elements should be defined. In elastic range, different simplified formulas which are mostly based on elementary structural mechanics are recommended in literature and codes. One of the common formulations for this purpose are equations (1-a) and (1-b) which are suggested in FEMA356 to calculate the in-plane stiffness of URM piers.

$$k = \frac{1}{\frac{h_{eff}^3}{3E_m I_g} + \frac{h_{eff}}{A_v G_m}} \quad \text{Piers free to rotate at top (Cantilever walls)} \tag{1-a}$$

$$k = \frac{1}{\frac{h_{eff}^3}{12E_m I_g} + \frac{h_{eff}}{A_v G_m}} \quad \text{Piers are prevented to rotate at bottom and top} \tag{1-b}$$

In these equations k is the in-plane lateral stiffness, h_{eff} is the effective height and A_v is the transverse shear area which all of the parameters belong to the URM piers. I_g is the moment of inertia of the pier section in flexural plane. Also, G_m and E_m are shear and elasticity modulus of the masonry material, respectively.

In inelastic range, the in-plane shear capacity and post yield force-displacement relationship for the URM piers are required. The general mohr-coulomb relationship (Equation 2) is recommended by different codes like Eurocode6 and FEMA356 to calculate the in-plane shear capacity of URM walls.

$$\tau = C + \sigma\mu \tag{2}$$

In equation (2), τ is the shear stress which causes sliding failure in mortar bedjoints, C is the brick-mortar bond shear cohesion, σ is the normal stress on mortar bedjoints and μ is the brick-mortar bond coefficient of friction.

In Eurocode6 the in-plane shear capacity of URM walls with filled and unfilled mortar head-joints is calculated by equations (3-a) and (3-b), respectively.

$$f_{vk} = f_{vko} + 0.4\sigma_d \tag{3-a}$$

$$f_{vk} = 0.5f_{vko} + 0.4\sigma_d \tag{3-b}$$

Where f_{vk} is the characteristic shear strength of masonry, f_{vko} is the primary characteristic shear strength of masonry without compressive normal stress on bedjoints and σ_d stands for the compressive normal stress on bedjoints.

In FEMA356 the in-plane shear capacity of an URM wall corresponding to the sliding shear failure mode through mortar bedjoints is calculated by equation (4).

$$v_{me} = \frac{0.75\left(0.75v_{te} + \frac{P_{CE}}{A_n}\right)}{1.5} \tag{4}$$

In this equation, v_{me} is the expected shear strength of masonry, P_{CE} is the expected gravity load on the masonry pier, A_n is the net transverse area of the wall and v_{te} is the shear cohesion strength of the mortar bedjoints.

According to equations (2) through (4), the in-plane shear capacity depends on the gravity loads on the wall.

Elastoplastic force-displacement relationship with or without hardening is recommended in literature and standards for inelastic lateral behaviour of the URM piers. For example the standard backbone curve for the plastic hinges in FEMA356 is illustrated in Fig.1-a. Some other researchers apply elastic-perfectly plastic relationship for the inelastic behaviour of URM piers. Akhaveissy (2012) applied elastic perfectly plastic relationship for post yield behaviour of URM piers, Fig 1-b. In current research elastoplastic beahviour is assumed for the post yield behaviour of URM piers.

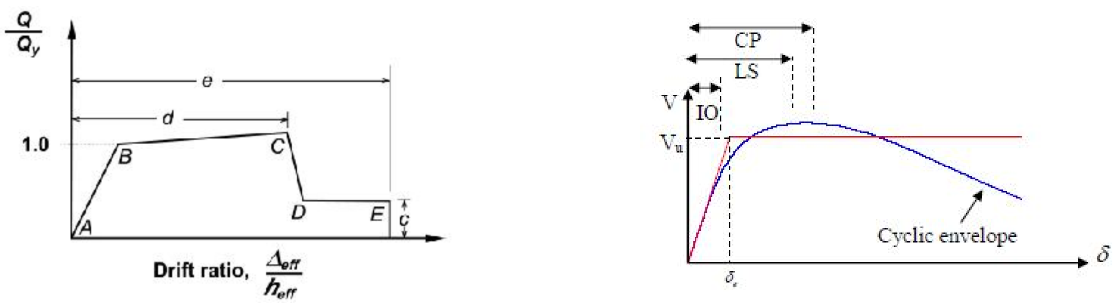

Figure 1. a- Standard plastic hinge for URM piers in FEMA 356 b- Elastoplastic load displacement curve used by Akhaveissy (2012)

EQIVALENT TRUSS MODEL FOR A SINGLE URM WALL

Here, an analogy is defined between an URM wall with a bi-element truss which is illustrated in Fig. 2. The truss consists of a vertical and an inclined element by an angle θ with horizontal direction. The vertical element is assumed rigid and the incline element is a compression only element. Whole of the gravity load as well as the lateral one are applied at joint b of the truss, separately.

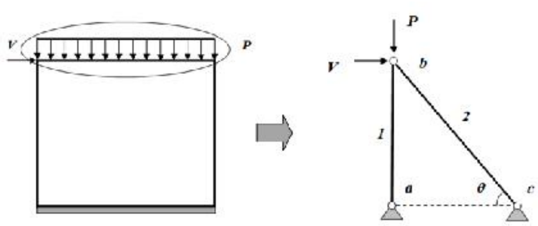

Figure 2. Equivalent truss model for an URM wall

Since the truss is a determinate structure, the internal forces are independent of stiffness of the elements. In small deformation, lateral stiffness of the truss depends only on stiffness of the inclined element as follows:

$$K_t = (\frac{AE}{l})_t \cos^2\theta \tag{5}$$

Where K_t is the truss lateral stiffness, E is the modulus of elasticity corresponding to the truss material, θ is the angle between inclined truss element and the horizontal direction and A and I are cross sectional area and length of the inclined truss element, respectively. Therefore, by application of equations (1) and (5), cross sectional area of the inclined element of the equivalent truss can be determined.

Mechanical and geometrical properties of the equivalent truss can be achieved by comparison of the internal forces in truss elements with equation 2 which estimates the in-plane shear capacity of URM walls. Equation 6 is the relationship between external loads and internal force in element#1 of the truss which is derived by a simple equilibrium equation.

$$V = \frac{F_1}{\tan\theta} + \frac{P}{\tan\theta} \tag{6}$$

In equation 6, F_1 is the axial force in element 1, P is the gravity load and V is the in-plane lateral load.

On the other hand equation 2 which defines the constitutive equation for the in-plane shear capacity of the URM walls can be rewritten as follows:

$$V_r = A_n\tau = A_nC + A_n\mu\sigma = A_nC + \mu P \tag{7}$$

Where V_r is the in-plane shear capacity of an URM wall and A_n is the transverse net area of the wall. Comparison of equations 6 and 7 indicates that these two equations have similar form. Therefore the parameters θ and F_1 corresponding to the truss model can be assessed as follows by a simple analogy:

$$F_1 = A_nC\tan\theta \tag{8}$$

$$\theta = \mathrm{Arc}\tan(\frac{1}{\mu}) \tag{98}$$

In this way the failure of the wall can be simulated by a plastic hinge in vertical element of the truss with capacity equal to F_1 obtained form equation 8. In summary the truss model with three characteristic properties which are appropriate tensile load-deflection relationship for the vertical element and cross sectional area and inclination angle of the inclined element, can simulate the nonlinear behaviour of an URM wall, Fig. 3.

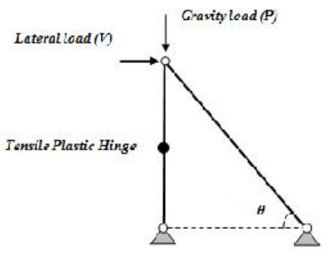

Figure 3. Definition of characteristic properties of equivalent truss model

EQIVALENT TRUSS MODEL FOR COMBINED URM WALLS

To perform a nonlinear static analysis on URM buildings, piers in plan and elevation should be combined. For this purpose at first, each pier in plan is modelled with an equivalent truss as explained previously. The portion of the gravity load on each pier is calculated and applied as a point load to the corresponding truss. For buildings with rigid diaphragms, top nodes of the trusses are constrained to displace equally in horizontal direction. This procedure is illustrated in fig. 4. Finally, a nonlinear push over analysis is performed on combination of piers and the push over curves are obtained. Performance of the building can be evaluated easily by the comparison of the lateral deformation of each pier by permissible values corresponding to each performance level.

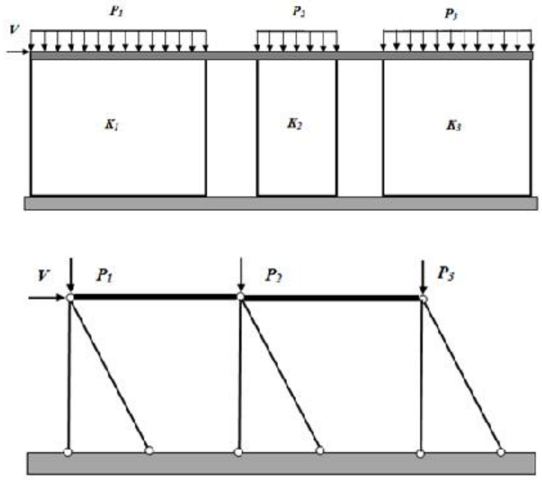

Figure 4. Equivalent truss model for combination of piers in plan

EXAMPLE

An URM wall with opening pattern as illustrated in fig. 5 is assumed. It consists of three piers with different lengths and effective heights. The boundary conditions of the piers are assumed according to the instructions in FEMA 356. The mechanical properties of the masonry are tabulated in Table. 1.

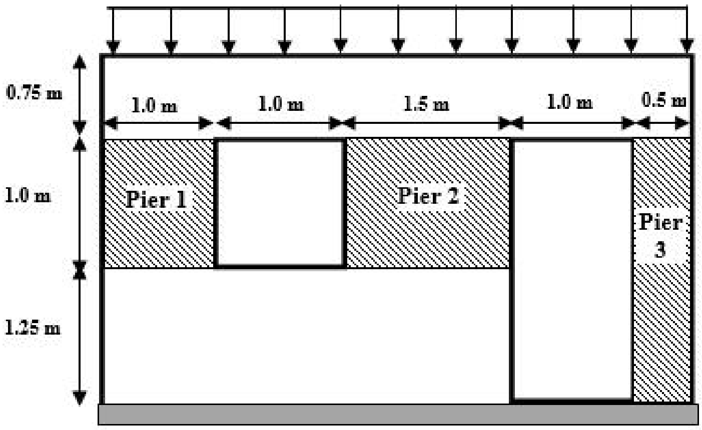

Figure 5. A Wall with three piers

At first stage, the stiffness of each pier is calculated with appropriate formulas (Equation. 1-b) according to their boundary conditions. Then, characteristic properties of the equivalent trusses such as plastic hinge capacity of the vertical element, cross sectional area and inclination angle of the inclined element for each equivalent truss are determined as explained before, Table 2. Finally, a nonlinear static analysis is performed on combination of the piers and the push over curve is obtained, Fig. 6. The results of the push over analysis indicate that the ultimate capacity of the wall is equal to 111.47. It is similar to the value which can be estimated by hand calculation. The push over curve also shows that the first yielding occurs in piers 1 and 3 simultaneously and finally the pier 2 yields. The performance of the piers can also be evaluated with comparison of the top lateral displacement of each truss with allowable values suggested in different codes.

Table 1- Material properties of masonry used for numerical studies of brick panels

Masonry Compressive Strength (MPa)	10.0
Modulus of Elasticity of Masonry (MPa)	5500.0
Shear Modulus of Masonry (MPa)	2200.0
Shear Cohesion of Brick-Mortar Bond (MPa)	0.1
Coefficient of Friction of Brick-Mortar Bond	0.75
Material Density (kN/m^3)	18.5

Table 2- Characteristic properties of equivalent trusses

	Pier 1	Pier 2	Pier 3
Inclination Angle (θ) (degree)	53.13	53.13	53.13
Vertical Element Length (mm)	3000.0	3000.0	3000.0
Inclined Element Cross Sectional Area (mm^2)	832278.0	1248423.0	208038.0
Modulus of Elasticity of Truss Material (MPa)	5500.0	5500.0	5500.0
Tensile Capacity of Vertical Element (kN)	26.64	39.96	13.32

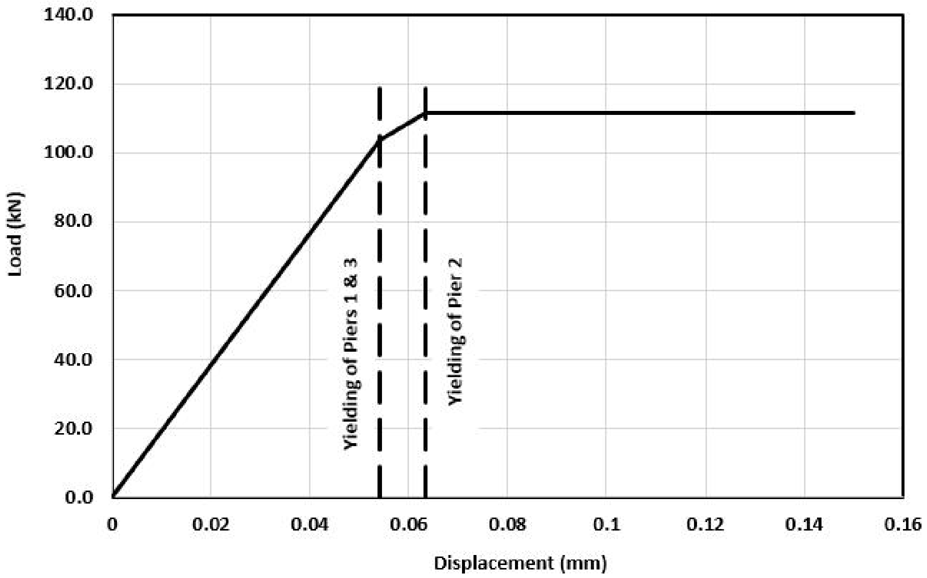

Figure 6. Push over curve of the wall obtained with equivalent truss method

EQIVALENT TRUSS MODEL FOR MULTISTORIES URM WALLS

In multi stories buildings the piers of the first story are modelled in a similar way to the method explained in previous section. For these piers the gravity loads which are inserted to the equivalent trusses are sum of the total gravity loads due to the above floors and walls. The gravity loads are applied as a point load on the top node of each equivalent truss.

The walls of other stories are simulated with tri element trusses as illustrated in Fig. 7. In each truss the vertical and inclined elements are the same as the truss model presented in previous section. Also, the mechanical and geometrical properties of these elements are determined in a similar way. The third element which is horizontal is rigid and constraints the bottom nodes of each truss to displace equally in horizontal direction. The equivalent trusses can displace freely in horizontal direction. Therefore roller supports are utilises for them. To enforce each floor level move equally in horizontal direction, the nodes of the trusses in any floor level are constrained to displace equally in horizontal direction. Similar to the first story, the gravity loads from above floors and walls are applied as a point load on the top node of each equivalent truss.

Finally, a nonlinear push over analysis with any lateral load pattern can be performed on combination of the walls. Also, the performance of the building can be evaluated simply by comparison of the lateral deformation of each pier by the permissible values for each performance level.

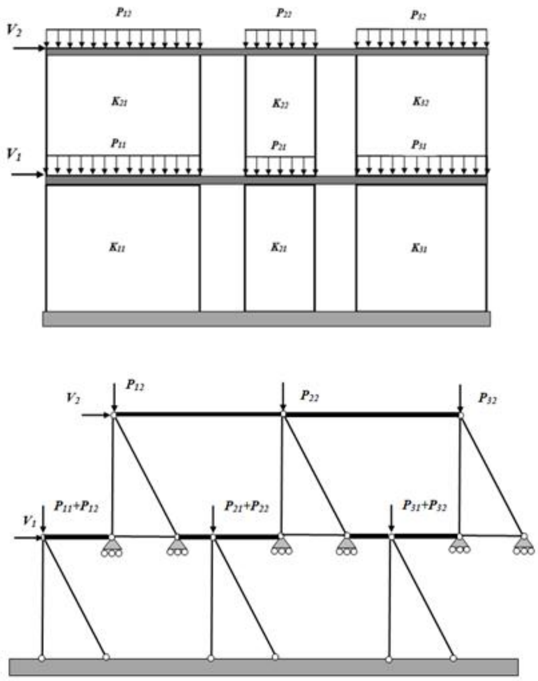

Figure 7. Equivalent truss model for combination of piers in elevation

EQIVALENT TRUSS MODEL FOR COMBINED URM WALLS

A two stories URM wall with opening pattern similar to the previous example is considered. It consists of three piers with different lengths and effective heights in each story. The mechanical properties of the masonry assumed here are tabulated in Table.1. The characteristic properties of the equivalent trusses are determined similar to the last example. Then the piers are combined in each story and elevation as explained before. Finally, a nonlinear static analysis with triangular lateral load pattern is performed on combination of the piers and the push over curve is obtained as shown in Fig. 8. The results indicate that the ultimate lateral capacity of the wall is equal to 152.76 kN. Also the performance of the piers can be evaluated with comparison of the top lateral displacement of each equivalent truss with allowable values suggested in different codes. Following the plastic hinges with increasing the lateral load shows that the pier 1 in first story and the piers 2 and 3 in second story yield simultaneously. Then the remaining piers yield at the ultimate capacity of the wall.

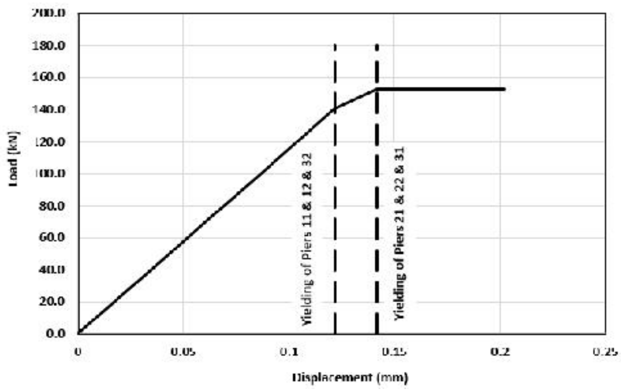

Figure 8. Push over curve of the 2 stories wall obtained with equivalent truss method

CONCLUSIONS

An equivalent truss model for nonlinear static analysis of unreinforced masonry buildings is presented in this article. The model can be used if sliding shear failure with Mohr-coulomb constitutive law is the dominant in-plane failure mode in the piers. In this method only truss elements with axial force and plastic hinges are used. This characteristic makes it a simple modelling approach and suitable for seismic evaluation of existing URM buildings. The main advantage of the current model to the other similar methods is that the effects of the gravity load on the in-plane capacity of the piers are considered automatically and there is no need to calculate the capacity of each pier separately for definition of the plastic hinge. Also, the model can be improved easily to consider other ductile failure modes of URM walls such as in-plane rocking failure mode in the piers. Also, results of the examples were solved by this method show that this modelling approach gives results with good precision.

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