A NEW OPTICAL METHOD FOR DETECTING SEISMIC VIBRATIONS

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ABSTRACT

In this paper we have described a new method to detecting characteristics of an oscillation system based on the Moiré technique. We can determine amplitude, resonance frequency and damping coefficient of an oscillation system both in vertical and horizontal direction. To do this approach, the displacements of oscillatory mass must be determined as possible as accurate. These displacements are recorded by Moiré detecting procedure. A spring-suspended mass (f0=10 Hz), whose position is monitored by Moiré technique, is used to testing this idea. Our detecting system consists of a pair of similar gratings which are installed near together without physical contact. The planes of the gratings are parallel and the lines of gratings have small angle respect together. Also, a laser diode (1 mW), a silicon photo-diode and a narrow slit have been used and fixed to the frame to illumination fringes movements due to the suspended mass movement. Due to moving the oscillatory mass and the fringes movements, the light intensity on the detector varies and is recorded as voltage. The output signal can be used to measure the oscillator characteristics. This method can detect displacements of the order of micron. Also, the experimental result and theoretical simulation are compared.

INTRODUCTION

Harmonic oscillation systems such as spring – mass or pendulum have varies applications. Today, some important geophysical instruments and sensors like seismometers, accelerometers and gravimeters based on the damped harmonic oscillators. Seismometers and accelerometers are instruments that record ground vibration during the propagation of elastic waves in the Earth and gravimeters are instruments that determine gravity acceleration. Therefore, having an appropriate readout system and the knowledge of characteristics of their oscillation system to processing and interpreting of data is very important. Furthermore, their amplitudes often are very small and difficult to detecting. In this work, we introduce a novel precise readout system that is based on the Moiré technique.

The Moiré technique has many applications in the measurement of very small displacements and light beam deflections. Moiré technique is based on the interference obtained when two transparent plates such as tow gratings are covered with equally period. If one of the plates is held over one another, they can be aligned so that no light will pass through or so that all light will pass through. Now, if one of the plates is placed over the other, and their lines have a small angle together, appear a new periodic structure that called Moiré pattern (Fig. 2). The dark and bright regions are called fringes. If the angle between the two gratings is increased, the separation between the light and dark fringes and the sensitivity decreases. By choosing a small angle the sensitivity and fringes separability increases. In this case, the period of moiré pattern dm is larger than the period of gratings *d* (Eq. 1). When one of the gratings moves d in perpendicular direction of grating's lines, it makes moiré pattern moves dm. Therefore the use of moiré technique magnifies the small displacements.

$$d_m = \frac{d}{2\sin(\theta/2)}. \tag{1}$$

Moiré readout system can measure amplitudes in the micron scale. In comparison with some techniques such as electromagnetic readout systems, our optical technique is free of EM noise, and the power of the output signal is easier to calibrate.

SETUP DESIGN

The vibration set up is a mass-spring oscillation system that consist of a mass suspended by two spring placed in top and bottom of the mass and fixed into a frame. The schematic diagram of our designed set up is shown in figure 1. We used the springs of a commercial geophone (SE-10) to build our moiré detecting system. The oscillator had a natural frequency of 10 Hz. We attached one Ronchi grating to the suspended mass and another one to the frame of the set up. The gratings are similar and have 20 Line per millimeter. If the angle between the lines of superimposed gratings is less than 6°, the dm/d ratio is larger than 10, which will result in a corresponding improvement in the measurement precision. The angle between the lines of superimposed gratings chosen is 6°. According to Eq. (1) dm is 0.48 mm. The gratings are installed close to each other without physical contact. They can move freely so the angle of the gratings lines remains constant. A laser diode (module size 8 mm×13 mm, wavelength 650 nm, input voltage 3 V, power 1 mW) was placed in front of the gratings, and a light-detector (silicon photodiode VTP1188s) faced the laser source from the opposite side. A narrow vertical slit (20 µm) was placed in front of the detector, and parallel to moiré fringes to narrow the light beam. The diode, the detector, and the slit were all fixed to the set up frame. The laser beam passes through the moiré pattern and narrow slit, before hitting on the light detector. The distance between the laser diode and the detector is 2 cm. The motion of the suspended mass results in a much larger motion of the Moiré patterns that is because of the displacement of one grating with respect to the other one. The Moiré fringes pass through the laser light and the intensity of the light varies as a result. The detector records these variations as a time series. The output of the light detector is electrical signal with amplitude which is proportional to the amplitude of suspended mass motion.

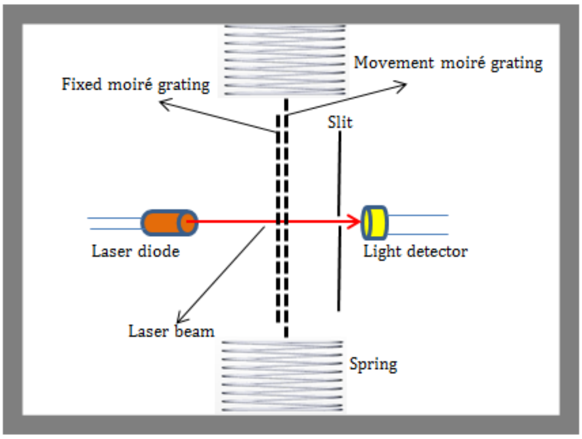

Figure. 1. Schematic diagram of the moiré detector system.

MATHEMATICAL FORMULATION

We present the mathematical formulation of the response of our detecting system to an excitation function. The transmittance coefficient of a Ronchi grating with lines perpendicular to x axis is given by:

$$i(x)=\sum_{-\infty}^{+\infty}a_n\exp(\frac{i2\pi nx}{d}), \quad (2)$$

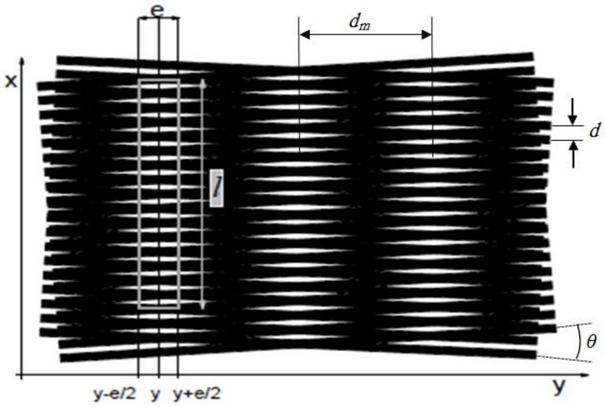

Figure 2. Moiré pattern obtained by superposition two Ronchi gratings that their lines have small angle(θ). A narrow slit with dimensions $l\times e$ is shown.

Where d is the period of grating, and a_n are the Fourier coefficients for a periodic structure. The transmittance coefficient for a superposition of two gratings with their lines placed at angles $\theta/2$ and $\theta/2$ to the y axes is given by:

$$I(x,y)=\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}a_na_m\exp(\frac{i2\pi}{d}[(n+m)x\cos(\frac{\theta}{2})+(n-m)x\sin(\frac{\theta}{2})]). \quad (3)$$

For n=-m, a periodic moiré pattern is formed whose period dm, is larger than d (Eq. 1). The average intensity of a light beam passing through a narrow slit of length l and negligible width e, placed in front of the moiré pattern (Figure 2), is given by:

$$\phi(y)=I_0\int_{y-\frac{e}{2}}^{y+\frac{e}{2}}dy\int_{-l/2}^{+l/2}T(x,y)dx, \quad (4)$$

Where I_0 is the intensity of the beam incident on the moiré pattern. Putting Eq. 2 in Eq. 4, we have:

$$\phi(y)=I_0\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}a_na_m\int_{y-\frac{e}{2}}^{y+\frac{e}{2}}\int_{-l/2}^{+l/2}\exp(\frac{i2\pi}{d}[(n+m)x\cos(\frac{\theta}{2})+(n-m)y\sin(\frac{\theta}{2})])dxdy \quad (5)$$

$$\phi(y)=I_0\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}a_na_m\int_{y-\frac{e}{2}}^{y+\frac{e}{2}}\exp(\frac{i2\pi}{d}(n-m)y\sin(\frac{\theta}{2}))dy\int_{-l/2}^{+l/2}\exp(\frac{i2\pi}{d}(n+m)x\cos(\frac{\theta}{2}))dx. \quad (6)$$

For n = -m, the value of the integral is l, and for n ≠ -m and large l it is negligible. Taking into account that the periodic structure is an even function, we obtain:

$$\phi(y)=I_0l\sum_{n=-\infty}^{+\infty}a_na_{-n}\int_{y-\frac{e}{2}}^{y+\frac{e}{2}}\exp(\frac{i2\pi ny}{d_m})dy \quad (7)$$

The real part of $\phi(y)$ is:

$$\phi(y)=I_0el\sum_{n=-\infty}^{+\infty}a_na_{-n}\sin c\left(\frac{\pi ne}{d_m}\right)\exp\left(\frac{i2n\pi y}{d_m}\right) \quad (8)$$

The structural function of Ronchi grating is even, therefore we have:

$$a_n=a_{-n}$$

The extrema of the above function are at:

$$\phi(y) = I_0 el \sum_{n=-\infty}^{+\infty} a_n^{\ 2} \sin c\left(\frac{\pi ne}{d_m}\right)\cos\left(\frac{2n\pi y}{d_m}\right)$$

Also, we have:

$$2(\phi_{\max} + \phi_{\min}) = I_0 l. \tag{9}$$

Now dividing (8) by (9), we obtain an expression for the average light intensity at position y and parallel to the moiré fringes:

$$\frac{\phi(y)}{2(\phi_{\max} + \phi_{\min})} = \frac{1}{4} + 2\sum_{n=1}^{+\infty} a_n^{\ 2} \sin c\left(\frac{\pi ne}{d_m}\right)\cos(\frac{2\pi ny}{d_m}). \tag{10}$$

The Moiré fringes have oscillatory movements with angular velocity ω and amplitude A with respect to the instrument frame. For an observer fixed to the fringes, the diode, the detector, and the slit have an oscillatory movement in the y-direction. The average intensity of a light beam through a slit of width e oscillating with angular velocity ω in front of the fringes is given by:

$$\bar{I}(t) = I_0 l\left[\frac{1}{4} + 2\sum_{n=1}^{+\infty} a_n^{\ 2} \sin c\left(\frac{\pi ne}{d_m}\right)\cos(\frac{2\pi n\left(A\exp(-\gamma t)\sin(\omega t+\varphi)\right)}{d_m})\right] \tag{11}$$

Where γ is the natural damping constant of the oscillator, t is time, and A is the amplitude of oscillation.

NUMERICAL SIMULATION

The numerical simulations have been presented based on the developed equations 10 and 12. Figure 3 shows the transmittance coefficient of the moiré pattern formed by a pair of Ronchi gratings (Eq. 10). Figure 4 shows the times series of the average intensity produced by the light detector (Eq. 11), as a result of oscillating Moiré fringes. We have considered the parameters, dm = 2.22 mm, e = 20 μm, ω = 20π (f = 10 Hz), and γ= 0.012. These parameters have been chosen as identical to those of the real instrument.

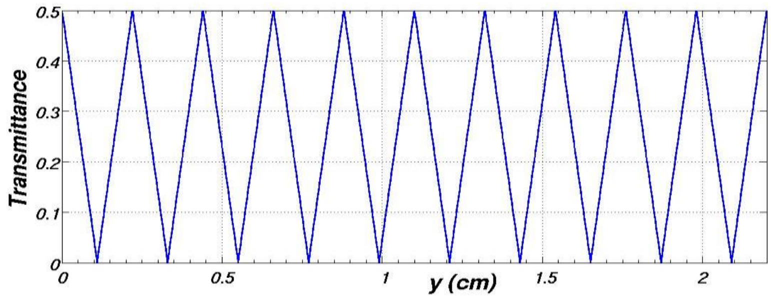

Figure 3. Ttransmittance coefficient of the moiré pattern formed by a pair of Ronchi gratings**.**

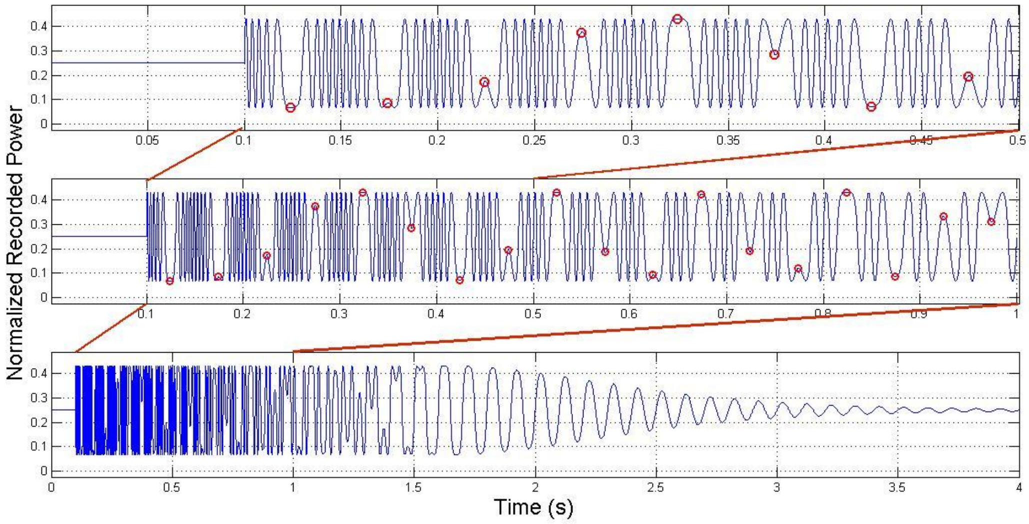

Figure 4. The times series of the average intensity produced by the light detector (Eq. 11). Red circles show the location of changing in phases in time series.

DETERMINATION OF THE OSCILLATION PARAMETERS

Due to an external stress on oscillation system, one of the gratings moves with respect to the other one. Therefore the moiré fringes pass through the laser light, and the power of light varies on the detector. The light power on the detector varies between two maximum and minimum values as a result of the bright and dark moiré fringes passing in front of the laser beam, respectively. The light power on the detector is triangular and periodic for a displacement larger than the gratings period. The measured power is linearly proportional to the displacement over a half period. Theoretical detail concerning linear behavior of the transmission function of the moiré pattern versus displacement can be found in [2]. Also, a similar theoretical investigation is reported in [5], where a method for submicron displacements measurement is suggested by measuring the autocorrelation of transmission function of a grating. The value of displacement of one of the gratings with respect to the other one is equal to the amplitude of the oscillations. In this case phase changing in time series identify oscillation phase changing. We can derive the amplitude of oscillations from the time series by:

$$A = (N + \delta)d \tag{12}$$

Where N is the number of complete period in time series occurs at first quadrate period of oscillation, δ is the fraction of one complete period that occurred, and d is the period of gratings that is $0.05mm$.We can derive δ by:

$$\delta = \frac{1}{2\pi}\sin^{-1}(\frac{V}{V_m}). \tag{13}$$

Where V_m is the value of maximum amplitude of output voltage due to passing the moiré fringes in front of the light detector, and V is the output voltage of detector at the end of variations of light intensity at first quadrate period of oscillation and before the first phase change in time series. Therefore, by this method we can determine the sequence amplitudes and then we can calculate the damping constant of the system. The uncertainty of measurements is 1 μm. Also, the power spectrum of time series shows the natural frequency of oscillation system.

EXPERIMENTAL RESULTS

We carried out an experiment to investigate the performance of our Moiré detecting system. We subjected our spring-suspended mass several times to a unit impulse produced by a controlled oscillator system. In this case, the spring-suspended mass has a natural damping constant with no other additional damping mechanism. Figure 5, shows an output of the Moiré system to a unit impulse (A=0.25 *mm*). A very good agreement can be observe between theory and experiment by comparison figure 4 and 5. The output shows that the amplitude of input pulls is equal to 5 moiré grating step that is 0.25 *mm* and by measuring two subsequence amplitudes damping constant of oscillation system can be determine. In this case that is 0.012. Also, Figure 6 shows the power spectrum of several output signals. As we can see, the 10 *Hz* component is clearly evident that shows the natural frequency of the oscillatory system.

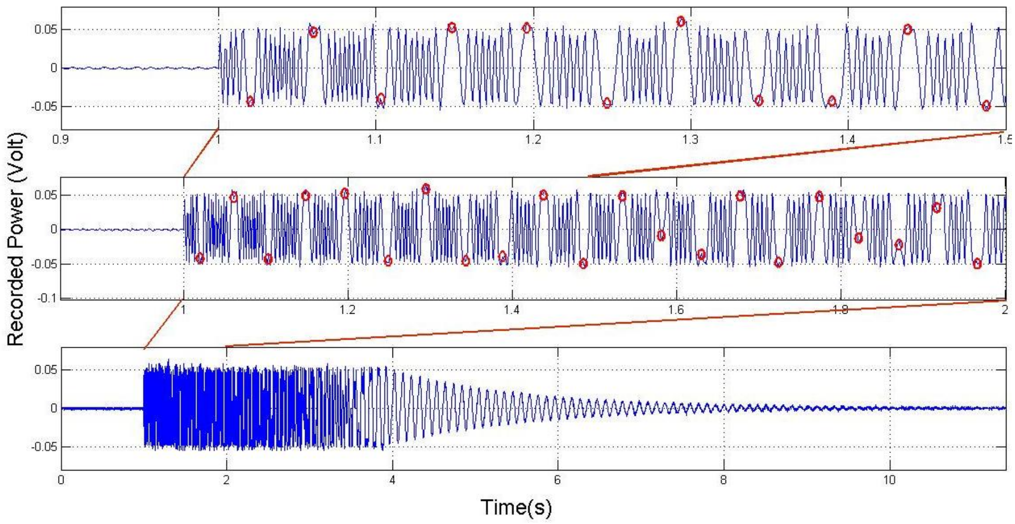

Figure 5. The output times series of moiré detecting system.

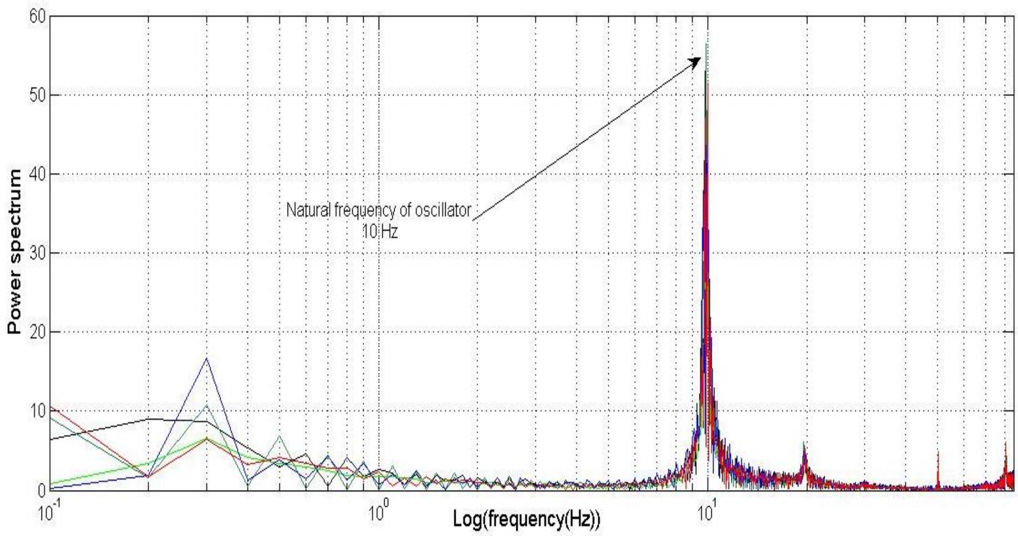

Figure 6. The power spectrum of several times series of moiré detecting system.
The 10 Hz pike is the natural frequency of oscillation system.

DISCUSSION AND CONCLUSION

In this paper we presented a new method to determine the parameters of oscillation systems. This detecting system employs the Moiré technique for its readout system. We showed that the actual response of our detecting system is in close agreement with its theoretical response. This system can detect micro oscillations and large oscillations, too. Also, measuring the actual amplitude of oscillations is possible. The minimum amplitude that is detectable by our set up is in order of micron. We can also, drive amplitude, natural frequency and damping factor of oscillation by using this detecting system. Furthermore, the moiré readout system has some advantages. For instance, its output is largely free of EM noise, whereas EM systems are vulnerable to various environmental noises, and the output can be easily regulated by altering the period and the angle of the gratings or the intensity of the light source. The applications of moiré detecting system vary from industrial vibration systems to geophysical studies and building vulnerable vibration sensors. Our instrument is easier to install, and insensitive to environmental conditions such as temperature fluctuations. Also, in the moiré detecting system, we can vary the sensitivity to detect the displacement by varying the gratings period, the angle between the rulings of the gratings. We can also, enhance the output of detector by enhancing the power of light source and enhancing the proportion of signal to noise.

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