EVALUATION OF LIQUEFACTION TRIGGERING UTILIZING PERTINENT UNCERTAINTIES

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Keywords: Liquefaction, Uncertainty, Reliability Analysis, Monte Carlo Simulation, Probabilistic Approach

ABSTRACT

Liquefaction analysis is one of the most challenging issues in seismic geotechnical engineering. The pertinent uncertainties involved in the evaluation of liquefaction such as heterogeneous nature of soil deposit and probabilistic nature of earthquake loading, make the phenomenon to be complicated. Evaluation of liquefaction includes deterministic and probabilistic methods. Deterministic methods are simple but they are not capable of considering uncertainties. Regarding to the pertinent uncertainties, it seems that reliability methods, which are based on statistics and probability theory, have a better estimation in comparison to deterministic methods. Reliability methods are able to consider the uncertainties and also to determine the appropriate safety factor proportional to variability of parameters and acceptable risk. In the present research reliability analysis of liquefaction utilizing Monte Carlo simulation has been studied. Application of the proposed method to the Loma Prieta earthquake cases verifies that deterministic method is not accurate enough and reliability analysis should be used instead.

INTRODUCTION

The requirements to design structures subjected to strong ground shaking have attracted the attention of many researchers. A designer should consider related problems of liquefaction from the safety and reliability point of view. The soil liquefaction phenomenon is an important issue of concern to earthquake geotechnical engineers in recent years. The liquefaction phenomenon happens when saturated granular media loses its shear strength due to the increase in pore water pressure under seismic loading. With the occurrence of this phenomenon, saturated sandy soils will lose their strength and soil particles will flow. This phenomenon has been observed in many earthquakes such as Alaska (1964), Niigata (1964), Loma Prieta (1989), Kobe (1995), Chi Chi (1999) and recently at Bushehr, Iran (2013).

Selection of geotechnical parameters is always one of the challenging issues for geotechnical engineers due to intrinsic uncertainties in soil and rock structures. Engineers usually apply large safety factors to cover the uncertainties which will lead to increase the project cost. It also could not be reliable enough since there is no explicit relation between factor of safety and probability of failure and it makes the engineering judgment to be complicated (Duncan, 2000). In recent years, probabilistic methods have been developed to overcome this deficiency.

In general, it is possible to categorize the geotechnical engineering uncertainties into two groups: inherent uncertainties and epistemic uncertainties (Griffiths & Fenton, 2007). Soil inherent uncertainties are due to the nature of variability of soil parameters in different locations and time. With regard to the nature of these uncertainties, the effect of them should not be neglected in geotechnical designs. The second category of uncertainties is due to the lack of information and knowledge in geotechnical engineering. Epistemic uncertainties include measurement errors, statistical uncertainties, and model uncertainties. It is possible to reduce the epistemic uncertainties by increasing the number of samples and observations. Regarding to the uncertainties in geotechnical engineering, it is not wise to consider soil parameters with deterministic values. Reliability analysis provides the opportunity to quantify the uncertainties (Jha & Suzuki, 2009). This approach can also determine the appropriate safety factor according to the variability of parameters and acceptable risk. Reliability analysis can be used as a supplementary tool to deterministic approach.

This paper is focused on the probability of liquefaction under dynamic loadings depending on variability of soil parameters using probabilistic analysis. The advantages and disadvantages of the deterministic and probabilistic analysis of liquefaction potential involving the related uncertainties of soil properties are discussed.

DETERMINISTIC ASSESSMENT OF LIQUEFACTION TRIGGERING

Seed & Idriss (1971) proposed a simplified method for liquefaction triggering. In the present paper, the most recent simplified method, which has been developed by Idriss & Boulanger (2010) is used. This method compares CSR (Cyclic Stress Ratio) and CRR (Cyclic Resistance Ratio) and determines the factor of safety against liquefaction potential. The liquefaction factor of safety (FS) is defined by the ratio of CRR to CSR based on Equation 1.

$$\mathrm{FS} = \frac{\mathrm{CRR}_{\mathrm{M_w},\sigma'_{v0}}}{\mathrm{CSR}} \tag{1}$$

in which $\mathrm{CRR}_{\mathrm{M_w},\sigma'_{v0}}$ is cyclic resistance ratio and CSR is cyclic stress ratio. If the FS<1, liquefaction occurs, and for the case of FS>1, it is safe (Kramer, 1996).

The CSR is defined as:

$$CSR = 0.65\left(\frac{a_{max}}{g}\right)\left(\frac{\sigma_{v0}}{\sigma'_{v0}}\right)\left(r_d\right) \tag{2}$$

where $\mathrm{CRR}_{\mathrm{M_w},\sigma'_{v0}}$ =cyclic resistance ratio, a_{max}= maximum horizontal acceleration, $\pm_{v0}$ =total stress, $\pm'_{v0}$ =effective stress and r_d=shear stress reduction factor.

According to Idriss & Boulanger (2010), r_d is defined in Equation 3.

$$r_d = \exp\left[\alpha + \beta M_w\right] \tag{3}$$

in which

$$\alpha = -1.012 - 1.126\sin\left[5.133 + \left(\frac{Z}{11.73}\right)\right] \tag{4}$$

$$\beta = 0.106 + 0.118\sin\left[5.142 + \left(\frac{Z}{11.28}\right)\right] \tag{5}$$

where Z is depth [m] and M_w is earthquake magnitude [Richter scale].

Cyclic resistance ratio (CRR) is usually correlated to an in-situ parameter such as SPT blow counts. Equation 6 has been proposed for calculating CRR. Also, it is proposed to use $(N_1)_{60cs}$ instead of $(N_1)_{60}$ to consider the effect of soil fine content.

$$\mathrm{CRR}_{M=7.5,\sigma'_{v0}=1atm} = \exp\left[\left(\frac{(N_1)_{60cs}}{14.1}\right) + \left(\frac{(N_1)_{60cs}}{126}\right)^2 - \left(\frac{(N_1)_{60cs}}{23.6}\right)^3 + \left(\frac{(N_1)_{60cs}}{25.4}\right)^4 - 2.8\right] \tag{6}$$

in which

$$(N_1)_{60cs} = (N_1)_{60} + \Delta(N_1)_{60} \tag{7}$$

$$\Delta(N_1)_{60} = \exp\left[1.63 + \left(\frac{9.7}{FC + 0.01}\right)\right] \tag{8}$$

$$(N_1)_{60} = C_N \times C_B \times C_E \times C_R \times C_S \times N \tag{9}$$

$$C_N = \left(\frac{P_a}{\sigma'_{v0}}\right)^{0.5} \leq 1.7 \tag{10}$$

where Pa=atmospheric pressure, $\pm'_{v0}$ =effective stress, FC=percent of fine content, C_N=overburden correction factor, C_B=borehole diameter correction factor, C_E=energy correction factor, C_R=rod correction factor, C_S=spoon correction factor and N=blow counts. Interested readers may refer to Idriss & Boulanger (2010) for more details.

The correlation of CRR is developed for a reference M_w=7.5, and $\pm'_{v0}$ =1 atmosphere and then adjusted to other conditions using Equation 11.

$$CRR_{M_w,\, \sigma'_{v0}} = CRR_{M_w=7.5, \sigma'_{v0}=1atm} \times MSF \times K_\sigma \times K_\alpha \tag{11}$$

where MSF is magnitude scaling factor, K_σ is overburden correction factor and $K_{\sim}$ is correction factor for sloping sites.

The magnitude scaling factor is used to account for the effect of duration of earthquake on liquefaction.

$$MSF = -0.058 + 6.9\exp\left(\frac{-M_w}{4}\right) \leq 1.8 \tag{12}$$

where M_w is magnitude of earthquake in Richter scale.

Equation 13 has been suggested take into account the effect of overburden effective stress such that

$$K_\sigma = 1 - C_\sigma \ln\left(\frac{\sigma'_{v0}}{P_a}\right) \leq 1.0 \tag{13}$$

in which

$$C_\sigma = \frac{1}{18.9\text{-}2.55\sqrt{(N_1)_{60cs}}} \tag{14}$$

where $\pm'_{v0}$ is effective stress and P_a is atmospheric pressure (100 kPa).

In order to consider the slope of the ground and the effect of driving shear stress, K_a is proposed. This coefficient is equal to one for horizontal soil stratum condition.

RELIABILITY ANSLYSIS AND MONTE CARLO SIMULATION (MCS)

Conventional design methods of geotechnical structures are usually based on limit state theory and factor of safety criterion. In these methods, all the parameters are defined deterministically. The factor of safety criterion makes engineering judgment complicated because there is no explicit relation between factor of safety and probability of events. Reliability analysis methods have been developed to overcome this deficiency. Reliability analysis provides the opportunity to consider the uncertainties quantitatively. In this approach, the factor of safety can be applied proportional to parameter uncertainties and the acceptable risk.

Reliability analysis methods are usually divided into three categories: analytical methods, approximate methods and simulation methods.

Simulation methods are among the accurate reliability methods. These methods predict the probability of event by simulating stochastic input parameters and implementing in repetitive calculations. In mathematics, these methods have been used for complex problems which their close-form solutions are not possible (i.e. large degrees of integration). Nowadays, regarding to rapid development of computer technology and available personal computer utilization of these methods has been increased in engineering problems. Monte Carlo simulation method is one of the most applicable methods of this category (Wang, 2012).

Monte Carlo Simulation (MCS) is a numerical process of repeatedly calculating a mathematical or empirical operator F(X) in which the variable $X=[x_1; x_2; \ldots ; x_n]$ within the operators are random or contain uncertainty with prescribed probability distributions (Ang and Tang, 2007). MCS is an accurate reliability analysis method and is applicable for any limit state approach (Phoon, 2008). It has been widely used in reliability analysis of geotechnical engineering problems such as slope stability, retaining walls, foundations and risk assessment of complex engineering problems. In the analysis process, stochastic values for each of the input parameter are selected based on its statistical parameters. The probability density function of stochastic parameters can have any shape but normal, log-normal and beta distribution functions are usually used based on the characteristics of the stochastic variables. These values are used to calculate performance function. This procedure is repeated for many times to obtain proper statistical distribution for performance function. Statistical analysis of this distribution enables the user to calculate the mean and standard deviation of performance function and finally to predict probability of events. Generally this method consists of four steps as follows (Phoon, 2008):

1. Choosing a stochastic value for each input variable according to assigned probability density function.
2. Calculating factor of safety using a proper deterministic analysis method based on selected values in step 1.
3. Repeating steps 1 and 2 for many times as required.
4. Determining distribution function for factor of safety and probability of failure.

The number of required Monte Carlo trials is dependent on the level of confidence in the solution and the amount of random variables. Statistically, Equation 15 has been recommended (Chandler, 1996):

$$N = \left(\frac{d^2}{4(1-£)^2} \right)^m \qquad (15)$$

where N= number of Monte Carlo trials, *d*= the normal standard deviation corresponding to the level of confidence, ε= desired level of confidence and m= number of random variables.

EVALUATION OF LIQUEFACTION TRIGGERING UTILIZING MONTE CARLO SIMULATION

Due to the probabilistic nature of earthquakes, it is required to analyze the response of earthquake loading by probabilistic methods. Generally, it is possible to divide the uncertainties of liquefaction into two categories: parameters uncertainties and model uncertainties. The effect of the parameter uncertainties has been studied in the present paper. For this purpose, the factor of safety function is selected as performance function and the parameters earthquake magnitude (M_w), maximum horizontal acceleration (a_{max}/g), total stress (σ_v), effective stress(σ'_v), fine content percent (FC) and SPT blow count (N_{SPT}) are chosen as stochastic variables according to the Tables 1 and 2. The statistical definition of the stochastic variables includes probability density function (PDF), coefficient of variation (COV) and correlation coefficient. The PDF and COV of variables are selected based on literature. Monte Carlo simulation method is based on generating stochastic values according to the stochastic parameters. If the stochastic parameters do not have appropriate correlation, generated stochastic values will affect the results. Generated stochastic values without/with considering correlation coefficient have been shown in Figures 1 and 2 respectively. Comparison of these figures demonstrates that considering correlation coefficient is necessary in Monte Carlo simulation. Figure 1 indicates that stochastic values do not follow a proper trend. For instance, the effective stress will exceed total stress in some cases and it is not reasonable.

The performance function is calculated using generated parameters for each trial. The probability of liquefaction is estimated by the probability density function of factor of safety. The probability of liquefaction (P_L) is equal to the area under the curve of probability density function with safety factors less than 1 as it is depicted in Figure 3. It is also possible to calculate it by Equation 16. In approximated methods, usually it is assumed that probability distribution function of factor of safety follows normal distribution but the results show that it is log normal distribution. The probability of liquefaction is classified according to Table 3. In this study, the above process is coded by MATLAB 7.

Table 1. Random parameters characteristics

Parameter	PDF	COV
N_{SPT}	Normal	20
a_{max}	Normal	5
M_w	Normal	5
$\pm_{v}$	Normal	15
$\pm'_{v}$	Normal	15
FC	Normal	10

Table 2. Correlation coefficient of parameters

Parameter	N_{SPT}	a_{max}	M_w	$\pm_{v0}$	$\pm'_{v0}$	FC
N_{SPT}	1	0	0	0	0	0
a_{max}	0	1	0.9	0	0	0
M_w	0	0.9	1	0	0	0
$\pm_{v0}$	0	0	0	1	0.9	0
$\pm'_{v0}$	0	0	0	0.9	1	0
FC	0	0	0	0	0	1

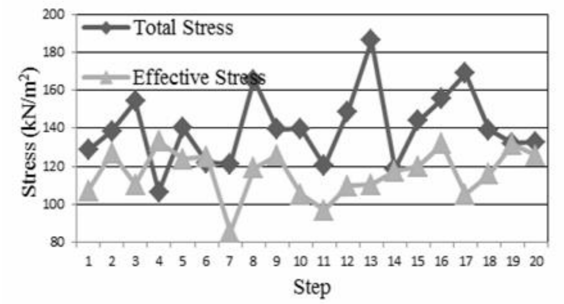

Figure 1. Generated random values without applying correlation coefficient.

Figure 2. Generated random values with applying correlation coefficient.

$$P_L = \frac{N_L}{N} \tag{16}$$

where: PL= probability of liquefaction, N_L= number of trials with safety factor less than 1, N= Total number of Monte Carlo trials.

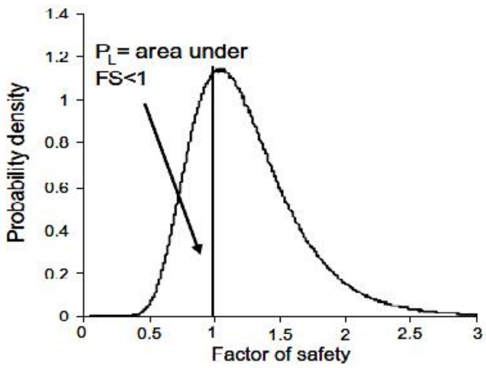

Figure 3. Probability density function and probability of liquefaction

Table 3. Classes of liquefaction potential

Class	Probability (%)	Description
1	P_L <15	Almost certain that it will not liquefy
2	15< P_L <35	Liquefaction unlikely
3	35< P_L <65	Liquefaction and non-liquefaction equally likely
2	65< P_L <85	Liquefaction very likely
5	85< P_L <100	Almost certain that it will liquefy

Regarding to the number of stochastic variables and the confidence level, 1.2 million of iterations have been done for simulations but it seems that large number of trials are not required. In order to investigate the effect of number of iterations on convergency of the results, a sensitivity analysis is examined. The results of MCS using 10,000 to 12 million of iterations have been shown in Figure 4. It demonstrates that the probability of liquefaction converges after about 500,000 of trials and this number of trials is enough for conventional required accuracy.

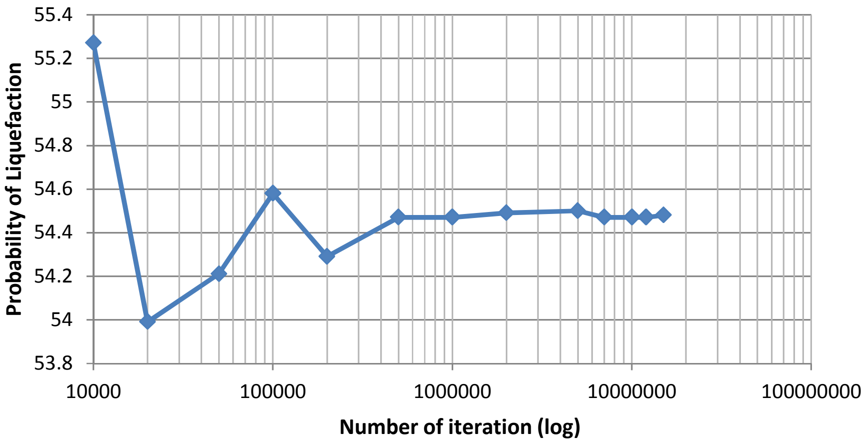

Figure 4. Effect of number of iterations on convergency of MCS results

VERIFICATION OF THE PROPOSED METHODOLOGY

To verify the application of the Monte Carlo Simulation method in liquefaction analysis, the Loma Prieta earthquake (October 1989), has been studied. Borehole specifications in different sites have been reported by Idriss & Boulanger (2010) and other researchers. Some parts of these data are presented in Table 4. It also gives the factor of safety with deterministic approach and the probability of liquefaction is estimated using Monte Carlo simulation. The statistical parameters for MCS are selected based on Tables 1 and 2.

Comparison between the results of deterministic approach and actual occurrence of liquefaction in this case study indicates that deterministic approach is not reliable to predict the liquefaction phenomenon. For example, in site numbers 13, 18, and 23 despite the fact that the factor of safety is more than 1, liquefaction had been occurred. On the other hand, in site numbers 9, 11, and 15, the factor of safety is less than 1, but there was no evidence of liquefaction occurrence. The results of Monte Carlo simulation indicate that this method has a good accuracy to predict liquefaction. It is observed that in site numbers 1, 3, 4, 8, 10, and 20 which probability of liquefaction is estimated less than 15%, liquefaction had not occurred. In site numbers 2, 12, 14, 21, and 22 which probability of liquefaction is estimated more than 85%, liquefaction had been observed. In other cases, the results are in agreement with Table 3 criteria. Estimating probability of liquefaction provides the opportunity to judge proportional to acceptable risk and it will facilitate engineering judgment in this issue.

Table 4. Evaluation of liquefaction utilizing deterministic method and Monte Carlo simulation –Loma Prieta earthquake (1989)

No.	Site	M_W	a_{max}/g	Z(m)	$\pm_{v0}{}^{*}$	$\pm'_{v0}{}^{*}$	N_{SPT}	FC(%)	Liquefy?	FS	P_L%
1	Alameda Bay	6.93	0.24	6.5	125	91	37	7	No	10.6	0
2	Faris Farm	6.93	0.37	6	106	92	9	8	Yes	0.56	99.44
3	General Fish	6.93	0.28	2.5	45	35	16.9	5	No	1.37	13.5
4	Hall Avenue	6.93	0.14	4.6	75	64	4.6	30	No	1.56	0.09
5	Marine Laboratory b1	6.93	0.28	4.6	87	65	11	3	Yes	0.77	84.19
6	Marine Laboratory b2	6.93	0.28	3.5	65	55	13	3	Yes	0.92	54.45
7	Marine Lab UBC-6-12	6.93	0.28	5.3	102	64	12	3	Yes	0.85	68.47
8	Marine No. 3 EB-1	6.93	0.28	2	35	35	18	1	No	1.96	1.5
9	Marine No. 3 EB-5	6.93	0.28	3.4	63	47	12	1	No	0.83	68.55
10	Mbari No. 4	6.93	0.28	3.4	62	48	18	5	No	1.31	14.5
11	Mbari Technology	6.93	0.28	3.4	62	48	12	4	No	0.86	66.35
12	Miler Farm CMF3	6.93	0.39	6.2	114	101	9.2	32	Yes	0.69	94.9
13	Miler Farm CMF5	6.93	0.39	7	130	108	20	13	Yes	1.03	47.9
14	Miler Farm CMF8	6.93	0.39	6	111	95	8.8	25	Yes	0.66	95.85
15	Miler Farm CMF10	6.93	0.39	8.4	158	105	19	20	No	0.93	55.9
16	Poo 7-2	6.93	0.28	6.3	121	89	14.4	3	Yes	0.8	77.8
17	Poo 7-3	6.93	0.28	6.3	121	89	16	3	Marginal	0.88	66.8
18	Por 2&3&4	6.93	0.18	5.9	97	73	4.3	50	Yes	1.07	27.8
19	Sandholtt UC-B10	6.93	0.28	3	55	43	9.5	2	Yes	0.89	60.5
20	Sandholtt UC-B10	6.93	0.28	6.1	115	73	26	5	No	6.83	1.87
21	SFOBB-1&2	6.93	0.27	6.3	118	86	7.5	8	Yes	0.6	98.75
22	State Beach UC-B1	6.93	0.28	3.4	61	46	6.3	1	Yes	0.64	97.2
23	State Beach UC-B2	6.93	0.28	4.9	90	67	12.8	1	Yes	1.04	38.38
24	Treasure Island	6.93	0.16	6.5	116	67	4.3	20	Yes	0.9474	51.9
25	Wood Marine UC_B4	6.93	0.28	1.8	32	25	6.7	35	Yes	0.85	65.2

*Unit=kN/m^2

CONCLUSIONS

Liquefaction potential is a probabilistic phenomenon due to probabilistic nature of earthquake and variability in soil deposits. Regarding to pertinent uncertainties, it seems that deterministic method is not suitable for liquefaction evaluation. On the other hand, the factor of safety criterion makes engineering judgment to be complicated because there is no explicit relation between factor of safety and probability of liquefaction. So reliability methods which consider uncertainties and estimate probability of liquefaction, facilitate engineering judgment. In the present paper reliability analysis of liquefaction triggering utilizing Monte Carlo simulation has been discussed. The application of proposed method shows that this method has enough accuracy for evaluation of liquefaction triggering and therefore this procedure is recommended for any other sites.

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