FLEXURAL STRENGTH AND MOMENT-CURVATURE CHARACTERISTICS OF SLENDER RECTANGULAR RC WALL SECTIONS

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ABSTRACT

Nonlinear static pushover analysis is used to assess seismic behaviour of structures reasonably well. Commercial structural analysis softwares currently available to perform pushover analysis (PoA) require definition of inelastic regions as idealised bilinear load-deformation response curves. Reinforced concrete (RC) structural walls in mid-rise buildings form the lateral load resisting system (LLRS). Thus, proper idealized moment-curvature response curves of these relatively slender RC walls form the key input for pushover analyses of such buildings. The idealized moment-curvature response curve must represent the cracked rigidity, flexural strength, and curvature ductility of the wall. Estimation of flexural strength and curvature ductility depends on significant strain levels in concrete and reinforcement bars at the onset of critical damage states. Yielding of extreme layer of tension reinforcement and compression failure in concrete are two such critical damage states of RC sections. A methodology to arrive at idealised moment-curvature curve of RC wall sections that can be used as an input to perform pushover analysis is proposed in the study.

Idealized moment-curvature curve, of deep RC wall sections with distributed longitudinal steel along its length, needs to consider yielding of an inner layer of longitudinal reinforcement as against yielding of the (extreme layer of) tension reinforcement in shallow beam sections, The paper presents a methodology for deep RC rectangular wall sections with distributed steel to first identify this critical *inner* layer of reinforcement below the centroidal axis (on tension side) based on energy balance of idealised moment-curvature curve with actual nonlinear curve, and then, to develop idealized moment-curvature response curve considering yielding of this inner layer of reinforcement representing the salient damage state. Further, the distance of the critical *inner* layer of tension reinforcement from highly compressed edge depends on percentage of longitudinal reinforcement bars in the section; the distance varies from 0.5D to 0.98D with increase in percentage of longitudinal reinforcement, where D is the length of the wall, but does not depend on plan aspect ratio of walls.

INTRODUCTION

RC walls have high flexural and shear strengths, and can be expected to efficiently resist strong earthquake shaking through controlled inelastic actions. Proper detailing of longitudinal (flexural) and transverse reinforcement helps in achieving strength and ductility. Two common ways of detailing flexural reinforcement in RC walls are: (a) uniform distribution of reinforcement along the length of the wall, and (b) uniform distribution of reinforcement along the length of the wall with additional reinforcement lumped at two ends of walls. But, in plane flexural strength of walls with the two distributions are almost identical, with enhanced shear strength in case of uniform distribution [Priestley, 2003]. Flexural strength and curvature ductility of rectangular RC wall sections can be determined from their nonlinear moment-curvature (M-φ) curves. In general, curvature φ of a RC section is defined as the ratio between strain at highly compressed edge to the depth of neutral axis, while ratio between ultimate curvature (φ_u) to yield curvature (φ_y) is termed as curvature ductility of the section. Curvature ductility is the general measure of ductile response of structure which significantly depends on the ultimate concrete compressive strain, compressive strength of concrete, yield strength of steel reinforcement, percentage of tension and compression reinforcement, and level of axial load.

Inelastic regions in select members are usually defined in the form of idealised *bilinear* moment-curvature curves (M-φ) to undertake PoA. The idealized M-φ response curve must represent the effective (cracked) rigidity, flexural strength, and curvature ductility of the wall section. Strain levels in concrete and steel at the onset of critical damage states like cracking of concrete, yielding of steel reinforcement in tension, and compression failure of concrete are used in the estimation of flexural strength and curvature ductility. This paper proposes a methodology to arrive at idealised bilinear M-φ curve of RC wall sections with uniformly distributed reinforcement along the length of the wall, at low levels of axial loads, that can be used as an input to perform pushover analysis.

ESTIMATION OF FLEXURAL STRENGTH AND CURVATURE

Flexural strength of RC walls can be estimated using principle of basic mechanics: equilibrium equation, compatibility conditions and constitutive relations for a RC wall section under flexure (Figure 1) can be written as (Eqs. 1, 2, 3 and 4):

Force equilibrium equation:

$$\Sigma\left(f_{sci} - f_{csci}\right)A_{sci} + f_{c,avg}\,b x_u - \Sigma f_{sti} A_{sti} = P \qquad (1)$$

Compatibility conditions:

$$\frac{\varepsilon_{sci}}{x_u - d''} = \frac{\varepsilon_c}{x_u} = \frac{\varepsilon_{sti}}{d - x_u} = \frac{\varepsilon_{csci}}{x_u - d''} \;; \qquad (2)$$

Constitutive relations:

$$f_{c,avg} = \begin{cases} 0.67 f_{ck}\left[\left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right) - \dfrac{1}{3}\left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right] & 0 \le \pounds_c \le \varepsilon_{co} \\ 0.67 f_{ck}\left[1 - \dfrac{1}{3}\left(\dfrac{\varepsilon_{co}}{\varepsilon_c}\right)\right] & \varepsilon_{co} < \pounds_c \le \varepsilon_{cu} \end{cases} \text{, and} \qquad (3)$$

$$f_{csc} = \begin{cases} 0.67 f_{ck}\left[2\left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right) - \left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right] & 0 \le \varepsilon_c \leqslant \varepsilon_{co} \\ 0.67 f_{ck} & \varepsilon_{co} < \varepsilon_c \leqslant \varepsilon_{cu} \end{cases} \qquad (4)$$

where A_{sti} is the area of reinforcement bars in layers under tension, A_{sci} the area of reinforcement bars in layers under compression, d'' the effective cover on compression side, $f_{c,avg}$ the average compressive stress in concrete, f_{sti} the stress in layers of reinforcement bars under tension and f_{sci} the stress in layers of reinforcement bars under compression (both estimated from stress-strain characteristics of reinforcement bar), x_u the depth of neutral axis, f_{ck} the characteristic strength of concrete, ε_c the compressive strain in concrete, ε_{co} the strain in concrete at highly compressed edge at peak stress, and ε_{cu} the ultimate strain in concrete at highly compressed edge at peak stress.

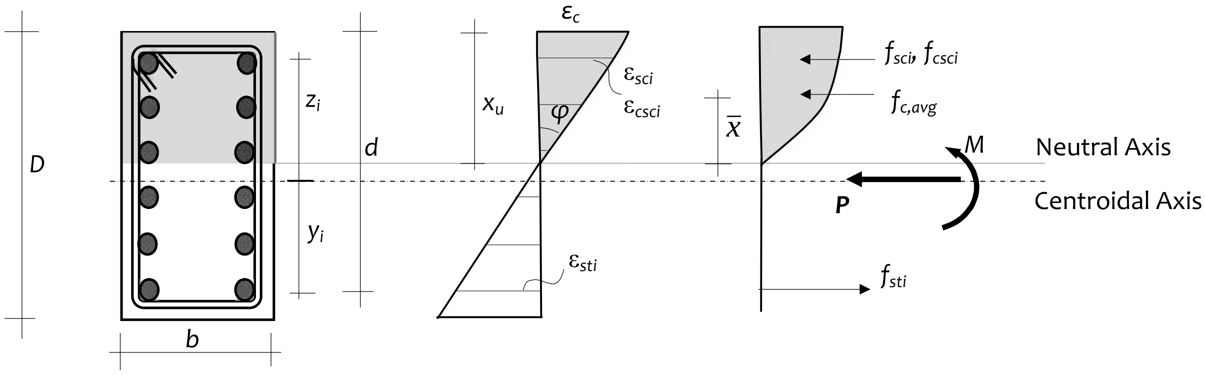

Figure 1: Typical strain and stress distributions across a rectangular RC section under flexure with salient geometric, strain and stress quantities

The depth of neutral axis is estimated through iterations of nonlinear constitutive relations. The curvature φ for a given strain distribution satisfying equilibrium Eq. (1) of a RC member (Figure 1) can be written as (Eq. (5)):

$$\varphi = \frac{\varepsilon_c}{x_u} = \frac{\varepsilon_{st}}{d - x_u} = \frac{\varepsilon_c + \varepsilon_{st}}{d} \tag{5}$$

Further, the associated moment can be estimated by considering moments of compressive and tensile forces about the centroidal axis as (Eq. (6)):

$$M = \sum f_{sti} A_{sti} y_i + \sum \left(f_{sci} - f_{csci}\right) A_{sci} z_i + f_{c,avg} b x_u \left(\frac{D}{2} - \left(x_u - \bar{x}\right)\right) \tag{6}$$

$$\text{where} \quad \bar{x} = \begin{cases} \left[\dfrac{\left[\dfrac{2}{3} - \dfrac{1}{4}\left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right)\right]}{\left[1 - \dfrac{1}{3}\left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right)\right]}\right] x_u & 0 \le \varepsilon_c \le \varepsilon_{co} \\ \left[\dfrac{\left[\dfrac{1}{2} - \dfrac{1}{12}\left(\dfrac{\varepsilon_{co}}{\varepsilon_c}\right)^2\right]}{\left[1 - \dfrac{1}{3}\left(\dfrac{\varepsilon_{co}}{\varepsilon_c}\right)\right]}\right] x_u & \varepsilon_{co} < \varepsilon_c \le \varepsilon_{cu} \end{cases} \tag{7}$$

y_i is the distance from centroid of layers of reinforcement bars in tension to centroidal axis, and z_i the distance from centroid of layers of reinforcement bars in compression to centroidal axis.

IDEALISED MOMENT-CURVATURE CURVE

The critical damage states called *limit states* of strain in concrete or reinforcing steel in the section, used to develop idealised *M-φ* curve of RC wall sections at low levels of axial loads are (Figure 2):

(1) *Cracking of concrete*: Maximum tensile strain ε_{cr} at the outermost edge of concrete reaches limiting tensile strain of concrete of 0.00008 [Hsu, 1993],

(2) *Yielding of critical layer of reinforcement on tension side*: Maximum tensile strain in the critical yielding layer of steel reinforcement bars reaches limiting strain ε_y of $\frac{f_y}{E_s} + 0.002$; and

(3) *Compression failure of concrete*: Maximum compressive strain at the highly compressed edge of concrete reaches limiting strain of ε_{cu}.

Due to large cross-sectional area, axial compressive load on RC wall sections are considerably smaller than that would cause a balanced failure condition [Park and Paulay, 1975]. Hence, a simple method is proposed using the above *limit states*, to develop idealized *M-φ* curves of rectangular RC slender wall sections subjected to low levels of axial loads (method for developing idealised *M-φ* curve at zero axial load of RC wall sections with reinforcement bars uniformly distributed along wall length is presented here). Also, the distance from highly compressed edge to critical inner layer of steel whose yielding has to be considered in arriving at a bilinear *M-φ* curve, closely matching in rigidity, strength and ductility, maintaining energy balance of nonlinear curve is also identified, for different percentages of steel and plan aspect ratio of wall sections.

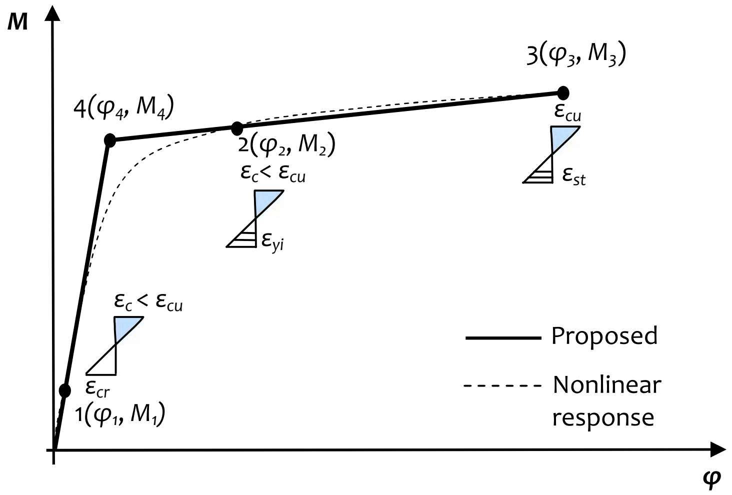

Figure 2: Proposed idealised *M-φ* curve of RC wall sections at low axial loads

Flexural strengths and curvatures at point 1 (cracking), 2 (yielding of an inner layer of reinforcement bars), and 3 (compression failure) are estimated as discussed previously corresponding to the strain variations at limit states shown in Figure 2. Point 4 is extrapolated from other three points and is given by Eqs. (8) and (9). The yielding layer of reinforcement bar to be considered to obtain point 2 is identified through numerical study.

$$\varphi_4 = \frac{M_2 - \left(\frac{M_3 - M_2}{\varphi_3 - \varphi_2}\right)\varphi_2}{\frac{M_1}{\varphi_1} - \left(\frac{M_3 - M_2}{\varphi_3 - \varphi_2}\right)} \tag{8}$$

$$M_4 = \frac{M_1}{\varphi_1}\varphi_4 \tag{9}$$

NUMERICAL STUDY

RC wall sections are considered of 150mm width and plan aspect ratio of 5, 6, 7, 8, 9, 10, 15, 20 and 25, with percentage of longitudinal reinforcement of 0.25, 0.50, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0, 2.25, 2.5, 2.75 and 3.0, uniformly distributed at 100mm spacing along the length of the wall. Grade of concrete and reinforcement bars used are M30 and Fe 415, respectively, as per Indian Standard [IS 456, 2000]. Strain limits are ε_{cr} = 0.00008, ε_{co} = 0.002, ε_{cu}= 0.0035 for concrete, and ε_y = 0.004075 for reinforcement bars. The idealised *M-φ* curves (at zero axial load, for select percentages of reinforcement bars of 0.25, 0.5, 0.75 and 1) are shown along with nonlinear *M-φ* curves in Figure 3. Here, two strategies are used to arrive at the idealised curves. In one, yielding of the extreme layer of reinforcement bars in tension is considered in arriving at point 2 (Figure 2). This leads to larger error in energy balance between the actual nonlinear curve and the idealised curve. Instead, using yielding of an inner layer of reinforcement bar, located at a distance x_y from the highly compressed edge, leads to more appropriate idealisation; the variation of the distance x_y of the critical layer normalised by the length *D* of the wall, for different percentage of longitudinal reinforcement is presented in Figure 4 and Table 1.

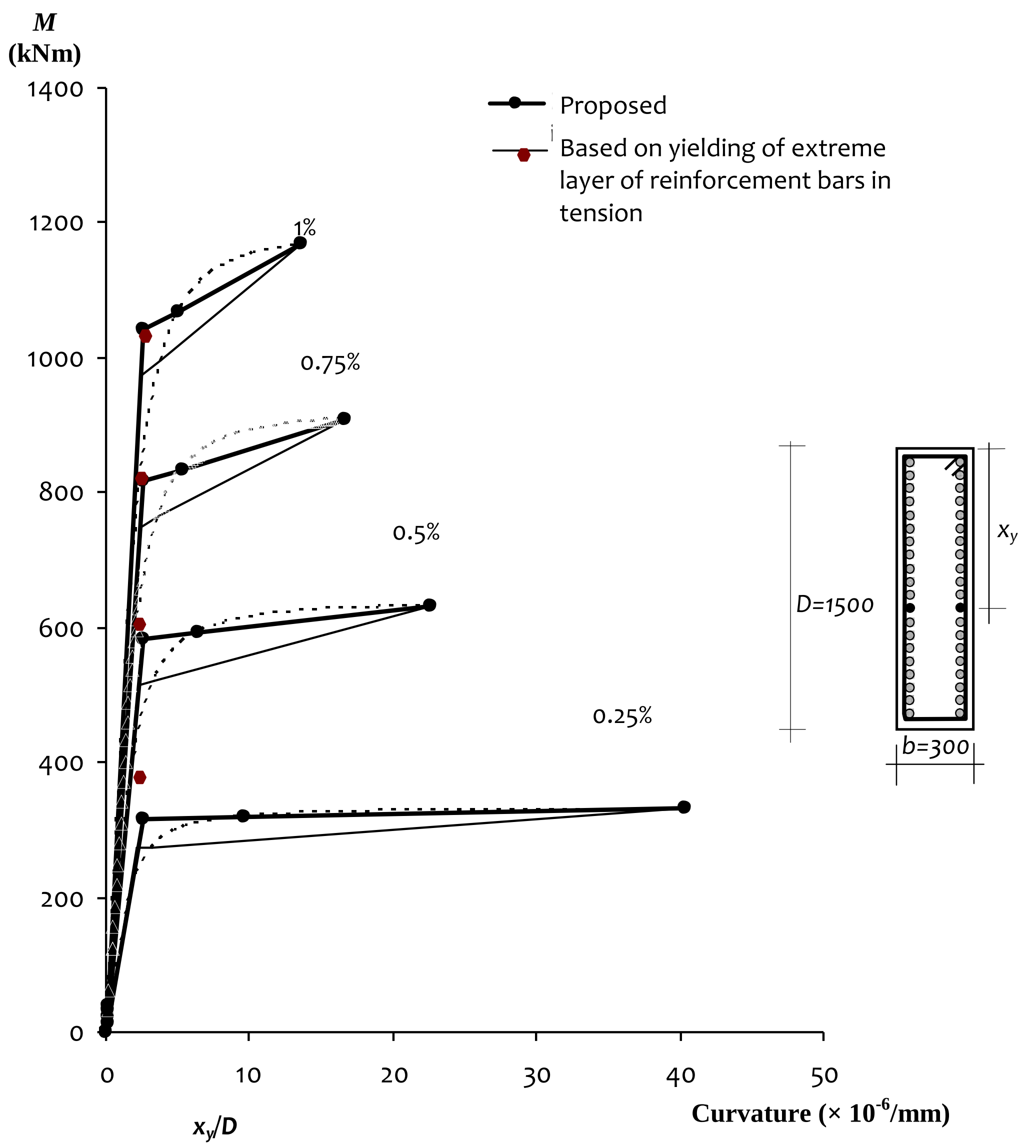

Figure 3: *M-φ* curves of example RC wall section of *D/b* 5 (percentages of reinforcement are shown)

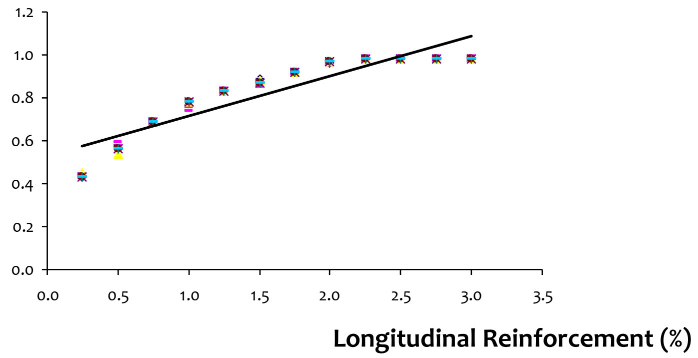

Figure 4: Variation of x_y/D with percentage of longitudinal reinforcement
(x_y/D shown for D/b 5, 6, 7, 8, 9, 10, 15, 20 and 25)

Table 1: Location of critical inner layer of reinforcement bars represented by x_y/D in rectangular RC wall sections with varying D/b and percentage of longitudinal reinforcement

	Longitudinal reinforcement (%)												
D/b	0.25	0.50	0.75	1.0	1.25	1.50	1.75	2.0	2.25	2.5	2.75	3.0	
5	0.43	0.56	0.69	0.76	0.83	0.89	0.92	0.96	0.96	0.98	0.98	0.98	
6	0.45	0.56	0.69	0.75	0.83	0.86	0.92	0.96	0.96	0.98	0.98	0.98	
7	0.45	0.54	0.69	0.78	0.83	0.88	0.92	0.97	0.97	0.98	0.98	0.98	
8	0.43	0.56	0.68	0.78	0.83	0.87	0.92	0.97	0.97	0.98	0.98	0.98	
9	0.43	0.56	0.69	0.78	0.84	0.87	0.92	0.97	0.97	0.98	0.98	0.98	x_y/D
10	0.43	0.56	0.69	0.78	0.84	0.87	0.92	0.97	0.97	0.98	0.98	0.98	
15	0.43	0.56	0.68	0.78	0.84	0.87	0.92	0.97	0.97	0.98	0.98	0.98	
20	0.43	0.56	0.69	0.78	0.84	0.87	0.92	0.97	0.97	0.98	0.98	0.98	
25	0.43	0.56	0.69	0.78	0.84	0.87	0.92	0.97	0.98	0.98	0.98	0.98	

The proposed idealised *M-φ* curve closely matches in initial rigidity, flexural strength, and curvature ductility, with actual nonlinear response curve maintaining energy balance, if yielding of an inner layer of reinforcement bars is considered. The distance x_y increases from 0.5*D* to 0.98*D* with increase in percentage of longitudinal reinforcement, but does not depend on the plan aspect ratio of walls. Also, this ratio x_y/D remains constant at higher percentages of reinforcement (more than 2%), when yielding of extreme layer of reinforcement bars in tension forms the basis to develop the idealised curve. This is because of dominant compression failure in concrete before yielding of majority of layers of reinforcement; this, in turn, reduces curvature ductility capacity of sections. Values of x_y/D given in Table 1 can be used as a guideline to identify the critical layer of reinforcement to be considered to develop idealised *M-φ* curve.

Finally, comparison is made of bilinear *M-φ* curves developed using the proposed method with actual nonlinear *M-φ* curves of a RC wall section for different percentages of reinforcement bars reported in literature [Cardenas and Magura, 1973]. Concrete of cylinder strength 42MPa, and reinforcement bars of yield strength 420MPa is used. Idealised *M-φ* curves obtained using the proposed methodology effectively represents rigidity, strength and curvature of the sections as represented by their actual nonlinear *M-φ* curves.

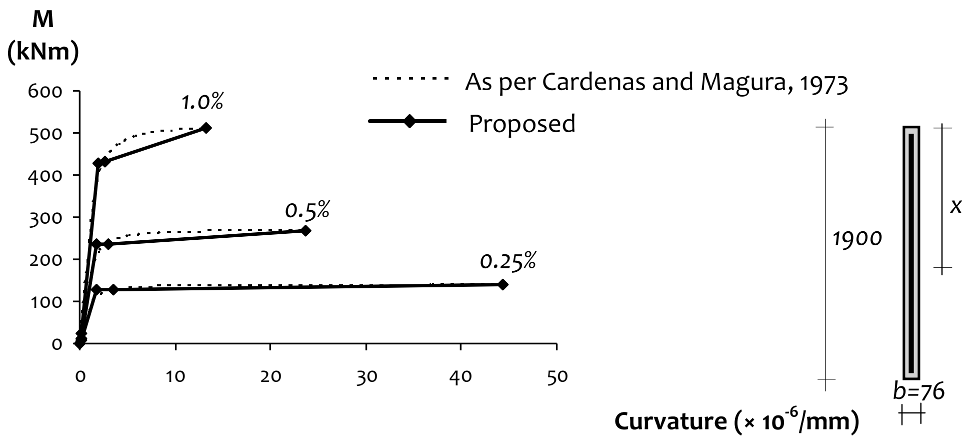

Figure 5: *M-φ* curves of example RC wall section [Cardenas and Magura, 1973]

CONCLUSIONS

The salient conclusions drawn from the study are:

(1) Simple hand-calculation based bilinear idealization proposed of actual nonlinear *M*-*φ* curves of rectangular RC wall sections effectively represents initial flexural rigidity, moment capacity and curvature ductility.

(2) The distance x_y of the critical *inner* layer of tension reinforcement from highly compressed edge to be considered in development of idealised bilinear moment-curvature curve of RC wall sections depends on percentage of longitudinal reinforcement bars in the section and varies from 0.5*D* at low percentage of reinforcement of 0.25% to 0.98*D* at high percentage of reinforcement of 3%, but does not vary with *D/b* of section.

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