SPATIOTEMPORAL CLUSTERING IN SIMPLE EARTHQUAKE FAULT SYSTEMS

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Keywords: Earthquake Modeling, Event Clustering, GR Scaling, Foreshocks, Aftershocks

ABSTRACT

While understanding the dynamics of seismic activity is fundamental to the investigation of the earthquake process, detailed studies of the earthquake fault system are difficult because the underlying dynamics of the system are not observable In addition, the fact that real earthquake fault systems are not composed of identical homogenous materials The variety of materials with different physical properties, such as frictional strength under pressure, can cause a variety of behaviors Inhomogeneities in the form of stress-relieving micro-cracks have been incorporated into long-range OFC models, resulting in a better understanding of GR scaling In addition, inhomogeneities have been introduced into fully elastic models resulting in either power-law statistics of event sizes or a separate distribution of events combined with large, system size events However, to date, none of these approaches have been able to reproduce both the temporal clustering and the complete magnitude-frequency distribution scaling regime that are primary features of natural seismicity and a critical component in the assessment of earthquake hazard In order to study a system with some aspects of spatial heterogeneities, we established a simple, long-range cellular automata model for earthquake fault systems based on the OFC model that incorporates a fixed percentage of stronger sites, or 'asperity cells', into the lattice These asperity sites are significantly stronger than the surrounding lattice sites but eventually rupture when the applied stress reaches their higher threshold stress The introduction of these spatial heterogeneities results in a rich array of spatial and temporal clustering in the model, including large, recurrent events with foreshock and aftershock sequences and accelerating seismic moment release and mimics those seen in natural fault systems along with GR scaling

INTRODUCTION

Despite the multitude of space-time patterns of activity observed in natural earthquake fault systems, the bulk of the research associated with these patterns has focused on a relatively small fraction of the events, those associated with either larger magnitudes or persistent, localized signals such as aftershock sequences

[Kanamori, 1981; Ogata, 1983; Utsu et al, 1995] One significant problem associated with studies of the earthquake fault network is that the underlying dynamics of the system are not observable [Herz and Hopfield, 1995; Rundle et al, 2000] In addition, the fact that nonlinear earthquake dynamics are coupled across a broad range of spatial and temporal scales [Kanamori, 1981; Main, 1996; Turcotte, 1997; Rundle et al, 1999; Scholz, 2002], combined with the occurrence of rare, extreme events and the associated patterns in seismic data [Schorlemmer and Gerstenberger, 2007; Vere-Jones, 1995, 2006; Zechar et al, 2010], means that computational simulations are critical to our understanding of the dynamics of the earthquake systems [see, eg, Rundle et al, 2003] Simple models of statistical fracture have been widely employed to test many of the typical assumptions and effective parameters inherent in the complicated dynamics of the earthquake fault system and their relative variability These models have been employed with remarkable success to advance our understanding of the statistical properties of earthquakes [Burridge and Knopoff, 1967; Otsuka, 1972; Rundle and Jackson, 1977; Rundle, 1988; Carlson and Langer, 1989; Nakanishi, 1990; Rundle and Brown, 1991; Olami et al, 1992; Klein et al, 1997; Klein et al, 2000; Alava et al 2006; Mori and Kawamura, 2008a, b] Burridge and Knopoff [1967] introduced a one-dimensional (1D) system of spring and blocks to study the role of friction along a fault in the propagation of an earthquake (Fig1-a) This model has been used extensively in the fields of geology, seismology, mechanical and materials engineering, mathematics, and physics (Vasconcelos, 1996, 1992; Clancy and Corcoran, 2005, 2006; Carlson and Langer, 1989; Carlson et al, 1991; Carlson et al 1994; Langer, 1992) After the initial BK studies, many other researchers investigated similar dynamical models of many-body systems with friction, ranging from propagation and rupture in earthquakes to the fracture of over layers on a rough substrate (Figure 1-b) Later Rundle and Brown [1991] presented a version with frictional sliding using the Mohr-Coulomb friction law that ignored inertial effects Olami, Feder and Christensen [1992] generalized Bak, Tang and Wiesenfeld [1987] sand-pile model and introduced a lattice version of the continuous, nonconservative cellular automata model (OFC) to investigate SOC behaviour in earthquakes However, most of these models included only short-range stress transfer None incorporated spatial heterogeneity into these earthquake-like fault models

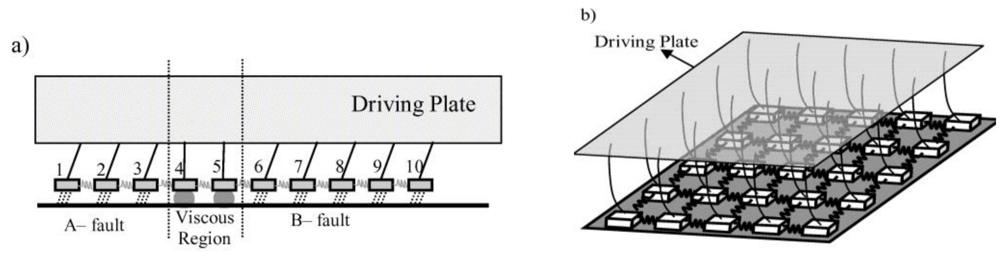

Figure 1 a) Schematic diagram of the BK numerical model (modified from Burridge and Knopoff, 1967) The geometry of a 2D spring block model (modified from Olami et al, 1992)

Because inhomogeneity plays an important role in the spatial and temporal behaviour of an earthquake fault [Serino et al, 2011; Dominguez et al, 2012, 2013], these models recently have been expanded to include different types of inhomogeneity, generally by varying individual parameters along the fault plane Several studies incorporated inhomogeneity into OFC models, although only for those with short-range or nearest neighbor stress transfer [Janosi and Kertesz, 1994; Torvund and Froyland, 1995; Ceva, 1995; Mousseau, 1996; Ramos et al 2006; Bach et al, 2008; Jagla, 2010] However, stress transfer in natural earthquake faults is elastic and, as a result, models with long-range interactions produce more realistic representations [Fisher et al, 1997; Ben Zion et al, 2008; Serino et al, 2011] OFC models with long-range stress transfer produce mean-field systems in stable or quasi-stable equilibrium, unlike short-range OFC models, and the existence and range of GR scaling is related to those periods of equilibrium [Gulbace et al, 2004; Klein et al, 2000, 2007; Rundle et al, 1995, 2000; Serino et al, 2011] In these mean-field systems near spinodal critical points, large events drive the system away from the critical point while small events bring it closer, resulting in GR scaling over several orders of magnitude

Serino et al [2011] incorporated damage inhomogeneities into the long-range OFC model in the form of stress relieving micro-cracks, resulting in a better understanding of the earthquake frequency-size distribution In subsequent work, Dominguez et al, [2012, 2013] incorporated spatial inhomogeneity into the lattice by clustering the dead sites in various patterns They found that the scaling depends not only on the amount of damage but also on the spatial distribution of that damage However, to date, none of the short or long-range models have been able to reproduce the variety of temporal clustering that is a primary feature of natural seismicity and a critical component in the assessment of earthquake hazard In this work, motivated by the structure of natural faults, we incorporate damage in the form of asperities into a simple cellular automata model for earthquake fault systems with long-range stress transfer These asperity sites fail less frequently than the regular sites, providing a time-dependent source and sink of stress, storing dissipated stress until asperity failure releases it back into the system The addition of structural asperities leaves the GR scaling intact but produces clustering of foreshocks and aftershocks as well as large quasi-periodic events

The model is a cellular automata version of earthquake faults based on the OFC [Olami et al, 1992] and RJB [Rundle and Jackson, 1977; Rundle and Brown, 1991] models with some minor variations Inhomogeneities are imposed on the model by inserting a percentage of either organized or randomly selected locations that accumulate higher levels of stress, similar to asperities on natural faults These sites are incorporated by varying the ability of these individual sites to support much higher levels of stress We observe a rich array of spatial and temporal clustering for the first time in these models, including large, recurrent events, seismic sequences consisting of foreshocks, mainshock and aftershocks, and accelerating moment release (AMR) In addition, we investigate the relationship between the spatial and temporal properties of the seismic sequences and the various parameters of the model, such as the overall stress dissipation and the percentage of asperities These statistics include the magnitude-frequency distribution scaling regime for the largest events, the relative activation of the foreshock and aftershock sequences, AMR and Thirumalai-Mountain (TM) metric fluctuations prior to the sequence mainshock and the Omori law for foreshocks and aftershocks

MODEL DYNAMICS

The model is a two-dimensional cellular automaton model with periodic boundary conditions In this model every site in the lattice is connected to z neighbors, which are defined as sites within a certain distance or stress interaction range, R A homogeneous residual stress σ_r is assigned to all the sites in the lattice To impose spatial inhomogeneity on the lattice, two sets of failure thresholds are introduced; ‘regular sites’ with a constant failure threshold of σ_f and ‘asperity sites’ with a much higher failure threshold ($\sigma_{f(asperity)}= \sigma_f+\Delta\sigma_f$) These asperity sites incorporate a percentage of stronger sites into the lattice that will support higher stress before failure

Initially, the internal stress variable, $\sigma_j(t)$, is randomly distributed on each site in such a way that the stress on all sites lies between the residual and failure stress thresholds ($\sigma_r<\sigma_i(t=0)<\sigma_f$) At t=0 no sites will have $\sigma_i > \sigma_f$ There are several ways to simulate the increase in stress associated with the dynamics of plate tectonics Here we use the so-called zero velocity limit [Olami et al, 1992] The entire lattice is searched for the site that minimizes (σ_f - σ_i) and that amount of stress is added to each site such that the stress on at least one site is now equal to its failure threshold That site fails and some fraction of its stress, given by α [σ_f – ($\sigma_r\pm\eta$)], is dissipated from the system α is a dissipation parameter ($0 < \alpha \leq 1$) which describes the portion of stress dissipated from the failed site and η is randomly distributed noise Stress on the failed site is lowered to ($\sigma_r\pm\eta$) and the remaining stress is distributed to its neighbors

After the first site failure, all neighbors, including asperity sites, are searched to determine if the stress change from the failed site caused any of others to reach their failure stress If so, the described procedure repeats for those neighbors and if not, the time step (known as the plate update) increases by unity and the lattice is searched again for the next site which minimizes (σ_f - σ_i) The size of each event is calculated from the total number of failures that expand from the first failed site during that plate update, or time step Unlike the original model, stress is dissipated from the system both at the regular lattice sites and through asperity sites which are placed inhomogeneously throughout the lattice The asperity sites fail less frequently than the regular sites and release much higher stress at the time of their failure resulting in inhomogeneous, time-dependent stress dissipation in this model

FREQUENCY DISTRIBUTION OF THE EVENTS

We compare our inhomogeneous model and a standard homogeneous model with no asperity sites in Figure 2 This figure shows time series ($6*10^5$ plate updates) and distribution of events (collected during 10^7 pu) for three different values of stress dissipation parameters α (Figures 2a, b, and c) The first diagram (i) in each set is the time series of events for the heterogeneous model with 1% of asperity sites The time steps in which an asperity site breaks are shown with a grey background shade The second diagram (ii) in each set is the time series for the homogeneous model with no asperity sites Figure 2d is the comparison between the frequency distributions for different values of α with and without asperity sites For the 1% asperity model the lattice does not break randomly in the time domain, despite the random spatial distribution of asperity sites The asperity model produces large, characteristic events that recur at constant intervals Those characteristic events occur less frequently as α, or stress dissipation, increases The distributions also confirm that, as stress dissipation increases, the largest events become smaller, as higher stress dissipation works to suppress large events (Serino et al, 2011) The frequency distributions also show that the model with 1% of asperities generates larger events compared to the homogeneous model

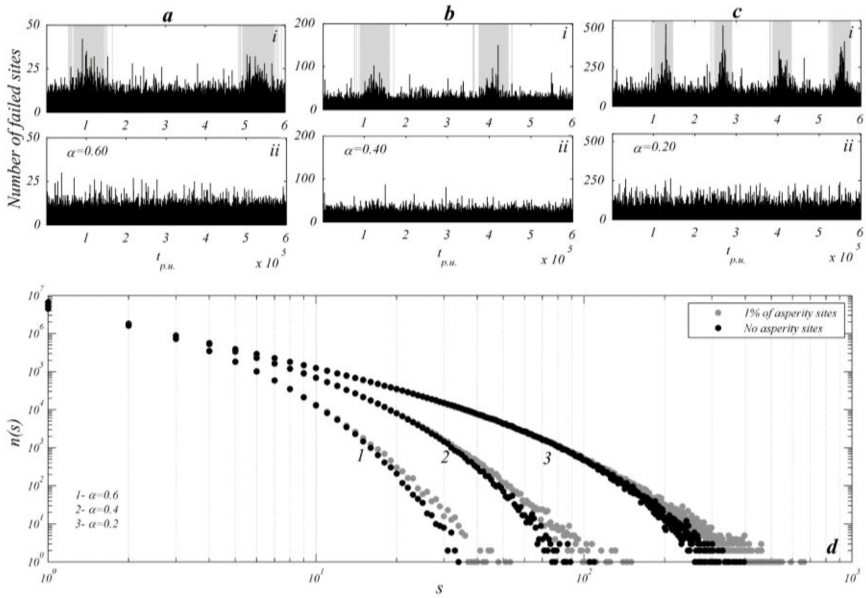

Figure 2 Time series of events during a period of $6*10^5$ pu for three different values of stress dissipation parameters (a) α = 06, (b) α = 04 and (c) α = 02; (i) are results for the model with 1% asperities (shaded background are those steps in which an asperity site breaks); (ii) are results for the homogeneous model (d) Comparison between the distribution of events for two different scenarios (with and without 1% of spatially random distributed asperity sites) for three values of stress dissipation parameter

The magnitude-frequency scaling of the long-range OFC model is studied in greater detail in order to investigate the effect of spatial inhomogeneities in earthquake fault-like systems with long-range stress transfer Serino et al [2011] studied an inhomogeneous version of OFC models with long-range stress transfer by adding random damage into the lattice and Dominguez et al, [2013] extended it by imposing various spatial configurations of damage into the model They studied various amounts of stress dissipation, α ($0<\alpha\leq1$) and showed that both stress dissipation and damage dissipation reduce the length of the scaling regime in the resulting magnitude-frequency distributions and reduce the size of the largest events

Here, we study the magnitude-frequency distribution of events in our inhomogeneous model for different percentages of asperity sites by changing the number of stronger sites in the lattice We find that the scaling relationship for the heterogeneous systems depends on the total amount of the asperity sites The results confirm previous findings that higher values of stress dissipation in the system decrease the length of the region in which the event sizes follow a scaling form To better understand this relationship, we extend this study by imposing a percentage of *randomly* distributed asperity sites into the lattice As expected, the magnitude-frequency event distribution confirms that as the percentage of asperities increases, the system produces significantly larger events (Figure 3a) However, the region in which the event sizes follow a scaling form becomes shorter and the relative number of moderate-sized events decreases as the number of asperities in the lattice is increased (Figure 3b) By increasing the number of asperities in the lattice, some of the moderate events appear to grow into a larger event This migration from the moderate to large sizes is the consequence of two effects When an asperity site breaks, a greater amount of stress is released into the system and that amount of released stress can cause the failure of more sites, especially in a system with long range stress transfer In addition, a greater number of randomly distributed asperity sites in the lattice increases the probability of asperity sites triggering each other A system with a higher density of asperities increases the chance that asperity blocks are inside the stress transfer range of a failing asperity That failure can result in a cascade behavior and a greater likelihood for a moderate-sized event to grow and become an extreme event

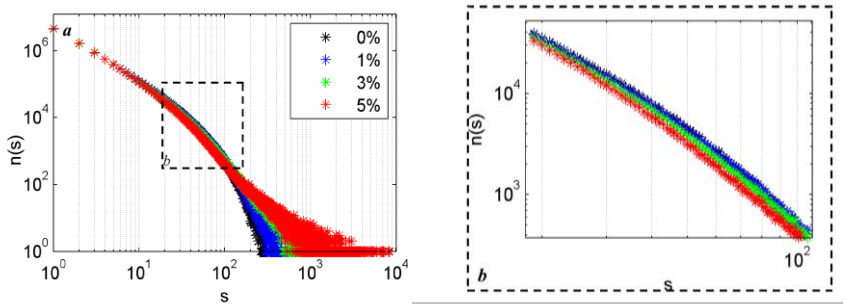

Figure 3 a) Magnitude-frequency distribution of events of size “s” for various amounts of randomly distributed asperity sites in a system with low stress dissipation, α=02; b) a closer view of the dashed box in a

ACCELERATED MOMENT RELEASE (AMR)

Mogi [1981] observed a regional increase in seismicity before great earthquakes, including an increase in the overall level of seismicity in the crust surrounding the future rupture zone, in conjunction with quiescence, or a relative shortage of events, along or near the fault Ellsworth et al [1981] also observed an increase in the rate of M5 events over a broad region in the years preceding the 1906 San Francisco earthquake This particular pattern of precursory seismicity appears to accelerate with the approach of the mainshock (AMR) [Bowman and King, 2001; Bowman et al, 1998; Sornette and Sammis, 1995; Bufe and Varnes, 1993; Sykes and Jaumé, 1990] and is defined by the equation

$$\varepsilon(t) = A + B(t_f - \mathrm{t})^m, \tag{1}$$

$\varepsilon(t)$ has been interpreted as either the accumulated seismic moment, the energy release or the Benioff strain release within a specified region, from some origin time t_0, up to time t

$$\varepsilon(t) = \sum_{1}^{N(t)} E_i^k, \tag{2}$$

is the number of events in the region between t_0 and t, E_i is the energy release from the i^{th} event, and k=0, 1/2, 1 A is a constant that depends on the background level of activity, t_f is the time of the mainshock, B is negative and m is a value between 03 and 07 Ben-Zion and Lyakhovsky [2002] analyzed the deformation preceding large earthquakes and obtained a 1-D analytical power-law time-to-failure relation for AMR before big events They found that phases of AMR exist when the seismicity occurring immediately before a large event has magnitude-frequency statistics over several ranges of magnitude These and similar results of Turcotte et al [2003] and Zoller et al [2006] are consistent with observed seismic activation before some large earthquakes

In this section, we investigate the AMR signal in the time series of the events in our model results Because the dynamics of our model requires that there is at least one broken site in every time step, we binned time into coarse-grained units of Δt=500 *pu* and counted the number of events greater than a predefined threshold in each bin Then we calculate the number of events greater than a chosen threshold in each coarse-grained time unit Figure 4 illustrates the cumulative number of events versus coarse-grained (binned) time for three different amounts of asperity sites in the models with different stress dissipation The increasing number of larger events prior to the mainshock in all the subfigures also produces an increasing rate of activity similar to AMR behavior before large events The results suggest that the AMR signal before the main event is more evident in those regions with more inhomogeneities and higher stress dissipation This is also the first time this phenomenon has been observed in any simple model in conjunction with GR scaling These results prompted investigation using the Thirumalai and Mountain (TM) metric [Thirumalai and Mountain, 1989; Tiampo et al 2003, 2007, 2010]

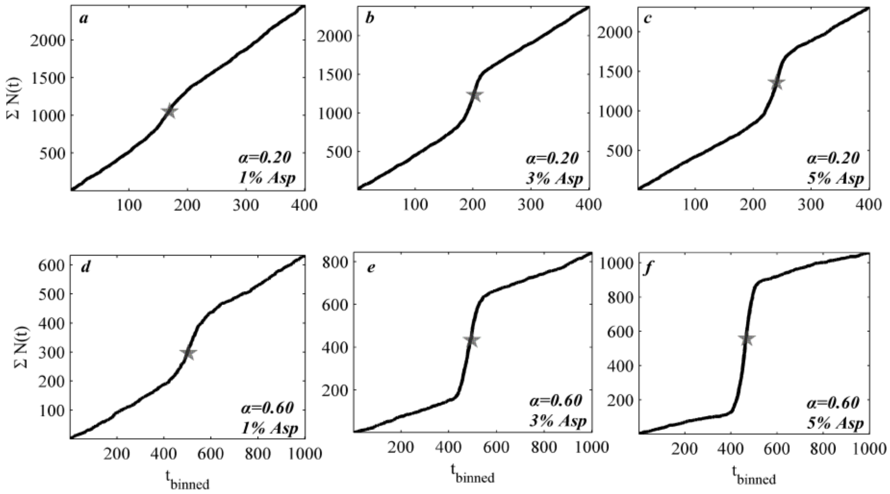

Figure 4 The accumulated number of events bigger than the defined threshold versus coarse-grained time for a-1% and α=020, b-3% and α=020, c-5% and α=020, d-1% and α=060, 3% and α=060 and f-5% and α=060 of randomly distributed asperity sites for two different stress dissipation parameters
The grey star on each graph shows the time of the main shock

TM METRIC BEHAVIOUR

In this section, we introduce the TM metric as a measure of event clustering The TM metric was introduced in the field of statistical physics of fluids to study the time scales necessary to achieve effective ergodicity in models of liquids and supercooled liquids [Thirumalai and Mountain, 1989] This method was first applied to earthquake simulations [Ferguson et al, 1999] and later was applied to regional seismicity by Tiampo et al [2003, 2007, 2010] They identified periods of metastable equilibrium in seismic activity, between large events, as well as the relationship between periods of effective ergodicity and certain types of seismicity patterns [Tiampo et al 2003, 2007, and 2010]

The TM metric measures effective ergodicity, or the difference between the time average of an observable (eg energy or stress) at each site and the ensemble average of that time average [Thirumalai et al, 1989; Mountain and Thirumalai, 1992; Thirumalai and Mountain, 1993] A necessary but not sufficient condition for ergodicity is stationarity, so that regions of phase space identified as effectively ergodic are maintaining stationary statistics over a given period of time In addition, it is a behavior generally limited to equilibrium states The TM metric is defined as

$$\Omega_x(t) \equiv \frac{1}{N}\sum_{j=1}^{N}\left(f_j(t)-\bar{f}(t)\right)^2, \tag{3}$$

where *j* refers to lattice sites, *N* is the total number of sites in the system

$$f_j(t) \equiv \frac{1}{t}\int_0^t x_j(t')dt', \tag{4}$$

and

$$\bar{f}(\mathrm{t}) \equiv \frac{1}{N}\sum_{j=1}^{N} f_j(\mathrm{t}), \tag{5}$$

and *x* is an observable quantity [Thirumalai and Mountain, 1989] The TM metric is the spatial variance of the temporal mean and should disappear by the law of large numbers in ergodic systems The system is “effectively ergodic” if

$$\Omega(\mathrm{t}) = \frac{1}{\mathrm{t}}, \mathrm{t} \to \infty \tag{6}$$

and the TM metric is used to determine whether or not a system is in statistical equilibrium

Here, we calculate the TM metric for our inhomogeneous fault model and use it to identify precursors, or foreshocks, of the mainshock in the associated time series Figure 5 shows the inverse TM metric plot for 1%, 3% and 5% percent of randomly distributed asperity sites in the low dissipation model The failure of the first asperity site in the series releases a large amount of stress into the system Because the system has long-range stress interactions and low stress dissipation, the released stress can migrate farther and trigger other asperity cells Figure 10 clearly shows the deviation of the linear inverse TM metric in those time steps prior to the mainshock (grey star) By increasing the total number of asperity sites from 1% (Figure 5a) to 5% (Figure 5c), the probability of triggering and therefore the amount of released stress before the mainshock increases and we can see stronger fluctuations in the TM metric plot Also, larger fluctuations in the TM metric plot occur prior to larger events in the time series Figure 6 is similar to Figure 5, but for higher stress dissipation (α=06) In this case, the probability of asperity triggering is very low and, as a result, the model produces smaller events compared to the low dissipation model The TM metric plot also shows smaller deviations for higher stress dissipation

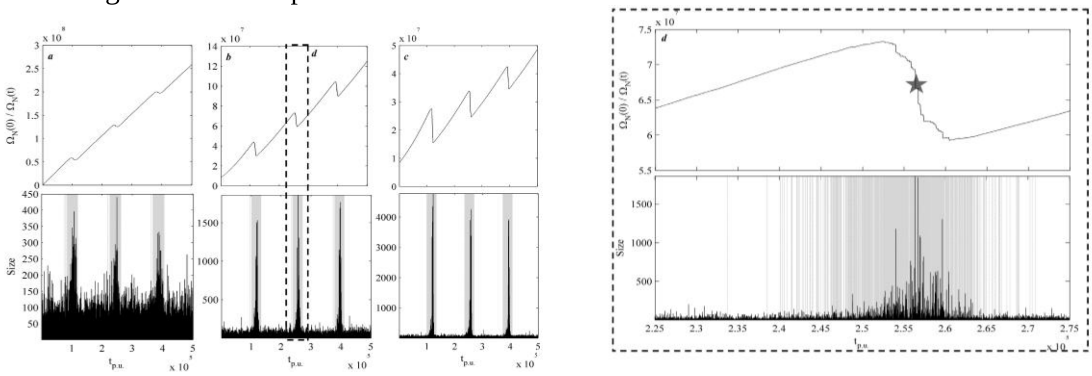

Figure 5 Inverse TM metric of each event for a-1%, b-3% and c-5% of randomly distributed asperity sites The grey star on the last graph shows the time of the main shock (All the above plots are for a model with α=02- low stress dissipation)

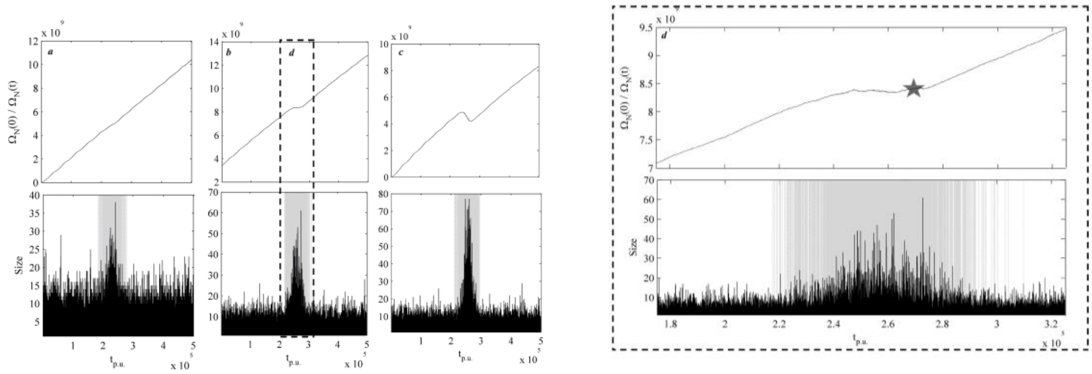

Figure 6 Inverse TM metric of each event for a-1%, b-3% and c-5% of randomly distributed asperity sites The grey star on the last graph shows the time of the main shock (All the above plots are for a model with α=06- high stress dissipation)

CONCLUSIONS

The inhomogeneity of natural materials with different physical properties in the Earth motivated this investigation of the effect of spatial inhomogeneity on the macroscopic properties of a many-body system The model studied here is a variation of the OFC and RJB cellular automata models of earthquake faults with long-range stress transfer In order to reproduce the spatial inhomogeneities of real earthquake faults in the model we have converted a percentage of selected locations in our lattice into local stress accumulators which have the ability to store and release a higher amount of stress than the surrounding lattice sites, similar to asperities on natural faults The initial results illustrate that increasing values of stress dissipation, regardless of the presence of inhomogeneity in the system, decrease the length of the scaling regime (Figure 2) In addition, the increasing number of asperity sites promotes the occurrence of larger events (Figure 3) The increasing number of larger events in these recurrent time series follows a pattern of precursory AMR activity prior to the mainshock comparable to that observed in natural seismicity (Figure 4) Plots of the inverse TM metric also show a clear deviation from stationarity before the mainshock (Figures 5 and 6)

This simple model supports the hypothesis that spatial heterogeneity has an impact on the primary features of both modeled and natural earthquake sequences and that the spatial and temporal patterns observed in natural seismicity may be controlled by the underlying physical properties The model reproduces the GR distribution and the size of the largest events and the length of the scaling regime on each fault are controlled by the amplitude of the stress dissipation of the system The superposition of many faults, each with a different amount of damage, results in GR scaling in the larger fault network Spatial features affect the critical point scaling behavior intrinsic to the earthquake fault process and the associated structure and geometry directly impacts the spatial and temporal nature of fault networks These findings suggest that asperities could be responsible for much of the spatial and temporal behavior of real earthquake fault systems and support the hypothesis that the patterns observed in natural seismicity may be controlled by the underlying physical complexity, rather than simple triggering alone

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