THE EFFECT OF SOIL PROPERTIES UNCERTAINTY ON THE GROUND MOTIONS INTENSITY

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ABSTRACT

Soil properties and soil depth to bedrock are parameters that can alter the ground motions' characteristics as they travel from depth to surface. The goal of this paper is to characterize the uncertainty of the soil properties and to propagate the effect of these uncertainties to the variability of ground motions intensity. It also investigates the effect of soil depth randomness on the ground motions intensity. A sensitivity analysis with the aid of Tornado diagram is performed in order to identify the most important soil parameter that affects the ground motions. Furthermore, two probabilistic analysis approaches, including Monte Carlo simulation technique (MC) and First Order Second Moment method (FOSM) are used for probabilistic analysis. Because of the time and cost consuming feature of full probabilistic analysis methods, such as MC, this study compares two methods to evaluate the accuracy of FOSM approximate method in the seismic response study of soil domain.

Performing nonlinear time history analysis for soil domain samples subjected to real earthquake records, the results of studies show that the soil depth can severely affect the ground motions intensity on the soil surface. The value of PGA of ground motions decreases as the soil depth increases. Soil shear modulus at small strains is recognized as the most effective parameter of soil that controls its dynamic behavior. It is also observed that stiffer soil experiences larger values of PGA. Finally, it's concluded that FOSM approximate method could be reasonably and effectively used instead of the MC simulation technique for evaluating the seismic response of soil domain regarding the uncertainty of soil properties.

INTRODUCTION

Consideration of uncertainties in the earthquake engineering problems is increasingly developing. Including the modeling parameters variability in the evaluation process of the engineering structures is very important since the performance of a structure at the time of earthquake is associated with the structure's characteristic parameters. Characteristically identifying the soil site on which the structure is established helps to precise evaluation and design of the structure because of the proved role of the soil-structure interaction in the structural behavior. However, the characteristic parameters of the soil site, which control its dynamic behavior when subjected to earthquake waves, are not carefully defined in most cases.

The uncertainty of a geotechnical problem can include soil properties uncertainty and soil depth to bedrock randomness. The inherent variability of soil because of being a natural material as well as soil identification sampling and testing methods can cause the uncertainty of its properties. Researchers have investigated the effects of these uncertainties on both the response of structures (Tang and Zhang, 2011; Rachowdhury, 2009; Na et al., 2009; Ray Chaudhury and Gupta, 2002) and the earthquake induced vibrations (Loperz and Modaressi, 2010; Badaoui et al., 2010; Marano et al., 2008; Bazzurro and Cornell, 2004). The depth of bedrock in a site varies from one place to another and changes the depth of the soil beneath the ground. This variability of depth can change the response of the soil on which the structure is established.

The present paper intends to investigate the seismic response of soil domain considering soil properties uncertainty and soil depth to bedrock randomness. To achieve this, the effect of such uncertainty is studied on the variability of ground motions intensity. Furthermore, the most effective soil parameter that affects PGA value is recognized. Besides, two probabilistic analytical approaches including MC simulation technique and FOSM analysis method for probabilistic analysis are complemented and their results are compared. For this, three different depths of soil medium including 10, 30 and 100 m are studied performing dynamic time history analysis using real earthquake records.

The seismic responses of the soil medium including amplification factor (AF), peak ground acceleration (PGA) and spectral responses of single degree of freedom oscillator (SDOF) on the soil surface are investigated. The soil medium is modeled using COM3, a finite element software developed at the University of Tokyo, Japan. The software simulates the soil profile using 8-node cubic elements known as Solid element. Solid element is able to assume the plain strain condition through which the infinity effects of the soil domain are taken into account. COM3 also can consider the homogeneity of soil medium and simulate the nonlinear mechanics of soil by defining its mechanical and physical characteristic parameters. This software provides a nonlinear model by which the nonlinear response of the soil domain subjected to earthquake excitation can be studied (Maekawa and Ishida, 2010; Maekawa et al., 2003).

GROUND MOTION DATABASE

20 real rock ground motion records from 8 different earthquakes worldwide were selected as the input seismic excitation for dynamic time history analysis of the soil domain. The records were selected based on moment magnitude of the event (M) and closest distance to the rupture zone (R) and were strong ground motions which belonged to the category of large magnitude (>6.5) with small distance to rupture zone (<30km), known as LMSR according to Shome and Cornell (1999).

The major horizontal component of each record was used as the input seismic excitation. Since we intended to study the value of PGA regarding different soil depths and in order to eliminate the record to record variability of initial peak acceleration, all records are scaled to a similar value of 0.35g. The accelerograms were directly applied to the interface of the bedrock and the soil medium in the models. Recording the upward traveling shear wave through the soil profile, the ground motion was obtained on the soil surface. Figure 1 displays the 5% damped acceleration response spectra of the selected earthquake records along with mean, median and mean ± σ curves and the scatter plot of the M and R of records. The standard deviation, σ, measures the amount of variation from the mean value.

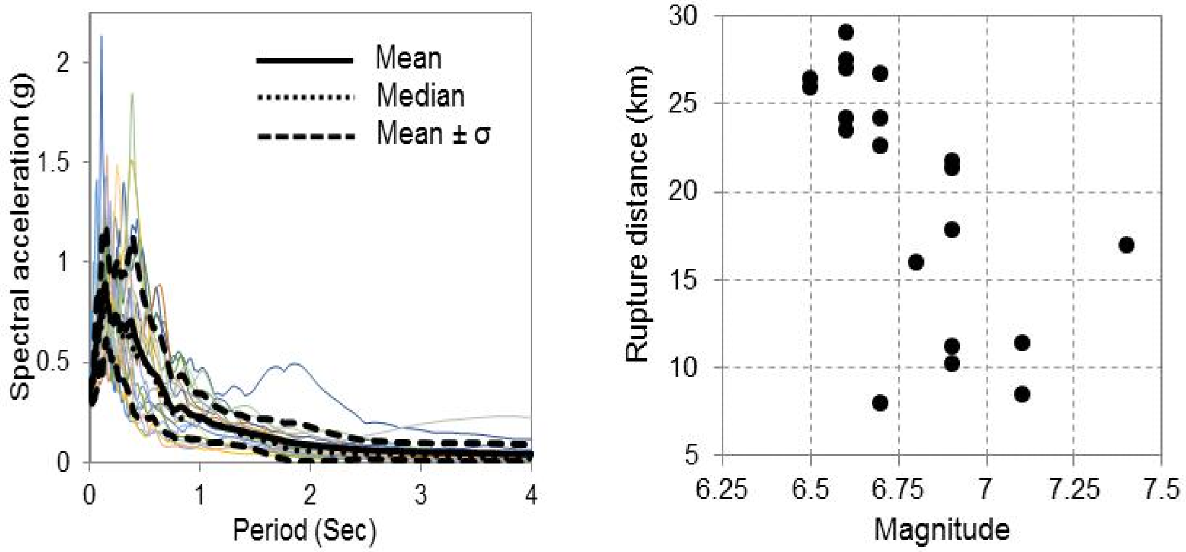

Figure 1. Characteristics of the input earthquake records

SOIL DOMAIN MODELING DESCRIPTION

The studied soil domain consists of single layer underlain by an elastic half-space bedrock and is cohessionless and relatively dense sandy deposit. Physical and mechanical properties of the soil are considered constant throughout the entire soil domain making it a homogenous medium. It's also assumed that the soil domain contains no water so the possibility of liquefaction is eliminated. As previously mentioned, soil domain is simulated by software COM3. In this software, soil stress-strain relationship is formulated by the multi-yield surface plasticity model. In this model, soil is idealized as an assembly of finite numbers of elasto-perfectly plastic components, which are conceptually connected in parallel. Since each component is given different yield strengths, all components subsequently begin to yield at different total shear strains, which results in a gradual increase of entire nonlinearity. The nonlinear behavior appears naturally as a combined response of all components. The general Drucker-Prager yield surface is implemented for each of the yield surfaces to represent the relation of the deviatoric shear stress and the mean stress (Maekawa et al., 2003).

The optimum domain length was defined large enough to consider the infinity effects in longitudinal direction. The thickness of the soil medium was assumed 1 m because of the existing plain strain condition in usage of the solid elements. The uniformity and stiffness of the bedrock were simulated by fixed nodes in the finite element model boundary zone at the bottom of the soil medium where connected to bedrock.

Five basic parameters of soil which control soil stiffness and strength properties including the maximum shear modulus at small strains, viscous damping ratio, relative density, material unit weight, and Poisson's ratio were defined as random variables. Mean and coefficient of variation (CV) of the soil parameters were determined according to the previous research works accomplished in laboratory and field regarding the same soil type as we considered (Kumar, 2008; Bazzurro and Cornell, 2004; Jones et al., 2002, Rieck and Houston, 2001; Darendeli, 2001; Kramer, 1996; Seed et al., 1984; Das, 1941).

The probability distribution of soil parameters according to the previous geotechnical studies is lognormal. The mean and coefficient of variation (CV) of the considered soil parameters are presented in Table 1. CV is defined as the ratio of the standard deviation to the mean value of a specific parameter. It is noteworthy that a perfect correlation was assumed between the soil shear modulus $G_{0,}$ and the friction angle φ, through Mohr-Coulomb theory and soil backbone curve. Hence, the variability of φ is not separately considered and its uncertainty was included within the soil G_0 uncertainty. The other parameters of the soil are considered independent of each other.

Table 1. Mean and coefficient of variation of soil properties

Soil properties	Symbol	Mean	CV (%)
Maximum shear modulus (kg/cm^2)	G_0	1700	50
Material unit weight (kg/cm^3)	γ	0.002	10
Relative density (%)	D_r	70	20
Poisson's ratio	v	0.35	20
Viscous damping ratio (%)	μ	2	25

In this study, the mean value of the soil properties is known as the Best Estimate value (BE). Considering the defined CV and BE value of each soil property, limit states of soil parameters including Upper Bound (UB) and Lower Bound (LB) are determined as below:

$$UB = (1 + CV) \times BE \tag{1}$$

$$LB = (1 - CV) \times BE \tag{2}$$

DEPTH RANDOMNESS STUDY RESULTS

In this study, three soil depths of 10, 30 and 100 m were studied in order to examine the effect of soil depth on the variability of ground motion characteristics that reach the soil surface after passing through different soil depths. To achieve this, soil samples were simulated having similar physical and mechanical characteristics but different depths. The soil properties were given their constant mean (BE) value according to Table 1. This assumption helps to eliminate the influence of soil parameters variability on its response so that the effect of the soil depth could exclusively be investigated.

In order to reduce the effect of earthquake record to record variability, the average result of 20 records is the basis of discussion. Hence, 20 accelerograms scaled on 0.35g were applied to the soil samples with different depths at the bedrock and recorded on the soil surface, then their average results were compared. The effect of soil depth on the response quantities of PGA, AF, and the SDOF spectral responses were studied and the mean and variability of each parameter were determined. The term AF is defined as the ratio of spectral acceleration of a specific record on the soil surface to the spectral acceleration of the same record at the bedrock.

Figure 2 shows the average AF and spectral responses of SDOF curves for 10, 30 and 100 m soil depths along with the average spectral responses of the input earthquake records. As displayed in Figure 2, the value of the maximum AF increased by increasing the soil depth, which indicates that the ground motion experiences more amplification while traveling from a greater depth to the surface. It's also observed that the maximum AF value occurred in the longer periods as the depth increased. It could be attributed to the long oscillation period of the greater soil depth which results in the resonance of the soil medium and earthquake input excitation. Furthermore, in the greater depth the number of peaks in AF curve increased indicating that the greater depth of soil experiences more resonances. The maximum AF value on average was equal to 2.1, 2.4 and 3.2 and occurred at the periods of 0.25, 0.6 and 1.6 s for 10, 30 and 100 m depths, respectively. According to the results, the maximum dispersion of AF occurred near the curve peak in all three depths. On the other hand, the maximum value of the spectral acceleration increased in soil depth of 100 m compared to 30 m and in 30 m compared to 10 m, while the value of maximum spectral velocity and displacement showed the inverse result. According to Mohraz (1976) three regions of amplifications are recognized in a response spectrum including the short-period or acceleration-sensitive region, medium-period or velocity-sensitive region and long-period or displacement-sensitive region. By examining the spectral responses, similar sensitive period ranges were observed such that the maximum value of spectral acceleration, velocity and displacement occurred in the shorter periods, respectively.

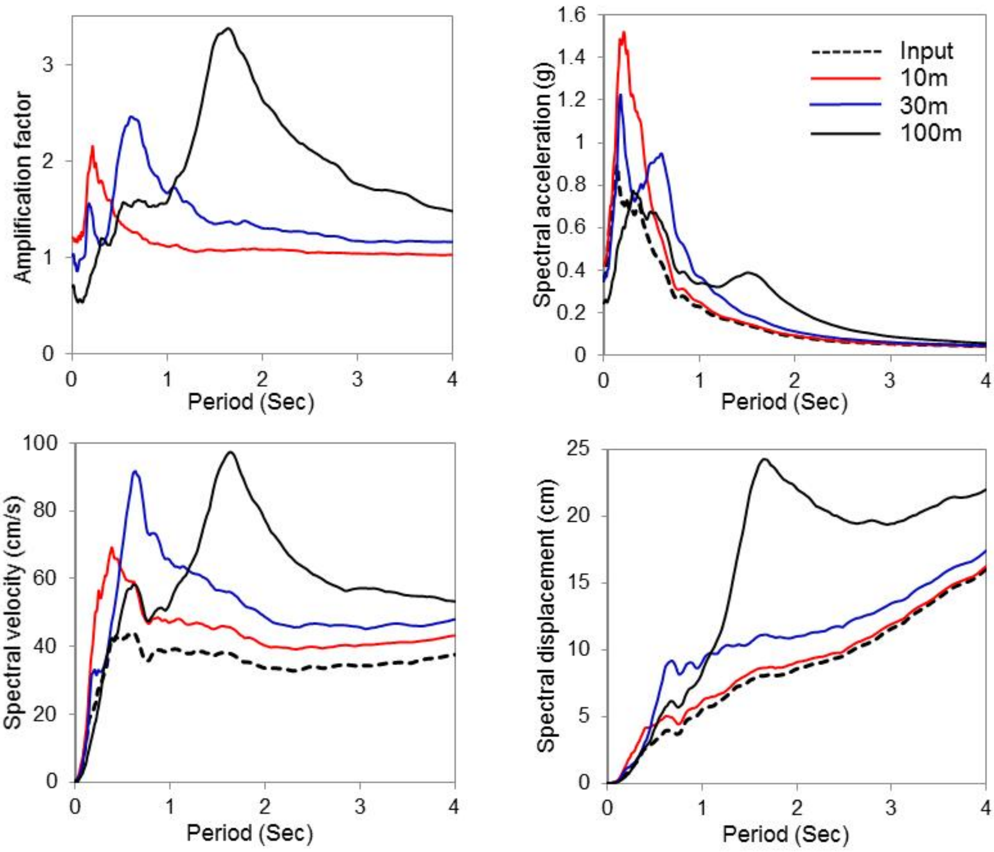

Figure 2. Comparison of amplification factor and spectral responses for 10, 30 and 100 m soil depth

The PGA is another parameter that intensely depends on the soil depth to bedrock. Figure 3 shows cumulative probability of the PGA occurrence based on lognormal distribution of the values. Smaller PGA values and larger dispersion of the results were observed at greater soil depths. The PGA values on average were equal to 0.42g, 0.36g and 0.24g on the soil surface and its CV values were equal to 0.12, 0.13 and 0.22 in 10, 30 and 100 m depth soil media, respectively.

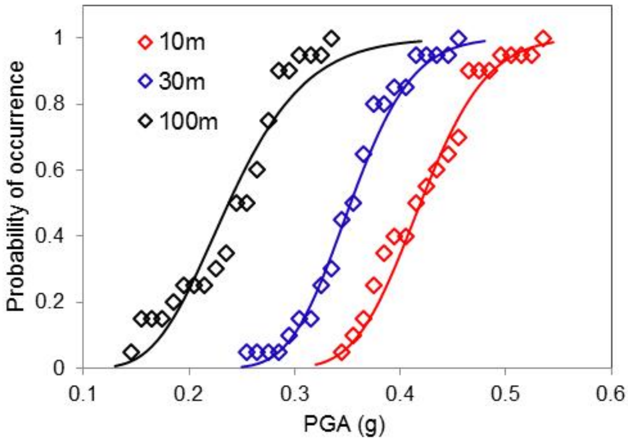

Figure 3. Cumulative probability of PGA occurrence for 10, 30 and 100 m soil depth

TORNADO DIAGRAM ANALYSIS

Tornado diagram analysis is used to compare the relative importance of the engaged uncertain variables. For each variable parameter, we need to estimate the lower, base and upper outcome. For this, only one soil parameter was modeled as variable at a time and the others were given their BE value. Doing this, the outcome sensitivity to each variable is determined individually. Tornado diagram consists of horizontal bars and each bar represents the sensitivity of the outcome to a particular variable. By calculating the difference between two lower and upper outcomes, length of each bar is determined. According to bars' length, they are arranged in descending order. The largest and the smallest bars show the most and least important parameters that affect the response.

The BE, LB and UB values of soil parameters were defined according to Table 1 and Eq. (1) and Eq. (2). The input seismic excitation was chosen based on the results obtained from the soil depth randomness study. The selected record was the one that showed the most similar result to the mean result obtained from averaging the results of 20 records. Moreover, PGA was chosen as the reference parameter for identifying the soil properties effect on the response.

The result of sensitivity analysis is displayed in Figure 4. The vertical solid line in the graphs belongs to the sample in which all of the parameters were given their BE value. The largest bar belongs to G_0 introducing it as the most effective soil parameter on the soil response. The other soil parameters were less important compared to the G_0.

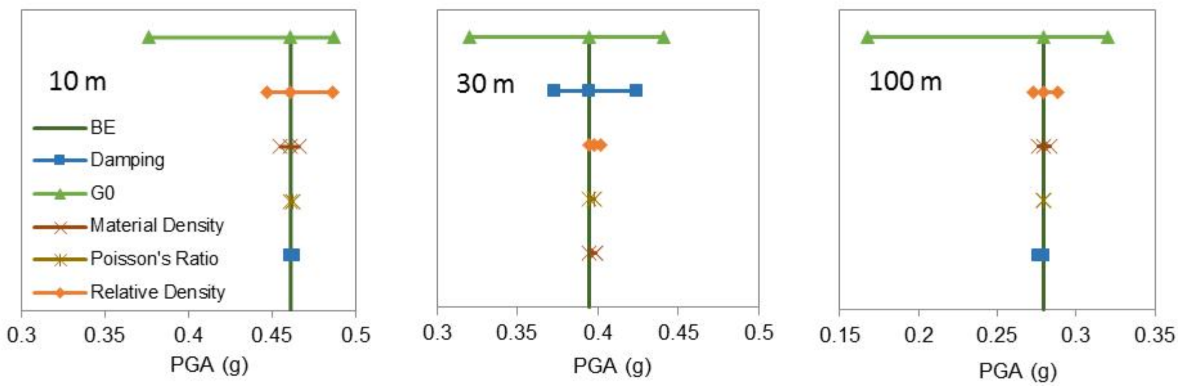

Figure 4. Tornado diagrams of sensitivity analysis results

PROBABILISTIC ANALYTICAL APPROACHES

MONTE CARLO SIMULATION TECHNIQUE

Monte Carlo simulation technique is an exact analytical approach for probabilistic analysis. MC consists of a set of computational algorithms based on repetitive random sampling for calculating the result. In this approach an uncertain problem is converted into problems with definite responses and the overall result is obtained by combining the results of substitute definite problems. The procedure is accomplished as follows: i) defining a set of possible inputs, ii) generating random values from the defined set, iii) performing deterministic analysis on the inputs, and iv) calculating the result. In this paper, the soil parameters were defined as the random variables. Employing MATLAB programming language, a simple random number generating code was utilized for generating the random inputs. The code generates random inputs based on the mean, σ and the probability distribution (lognormal in this study) of a desired parameter. The generated random inputs of soil parameters were used for simulation of the soil samples with different characteristics.

FOSM METHOD

First Order Second Moment is a statistical engineering analysis method on the basis of Taylor series expansion for calculating the response of an uncertain problem. In the FOSM method, the problem is defined as a variable-based function as Eq. (3), in which Y represents the uncertain problem and x is the varying input parameter. Ensuring the availability of the function derivatives at every point of x, the problem is solved by generating a relationship between the moments of the problem and the moments of input variable. This method is simple to use because obtaining two first moments is much simpler than exactly solving the problem. In this method, the mean and variance of the problem is determined based on the mean and variance of the input varying parameter. The parameter x in the function, is a random variable with the mean value of μ_x and variance of σ^2_x.The uncertain problem is rewritten as Eq. (4) on the basis of Taylor series expansion about the point x_0. The first moment of the function, μ_Y, represents the mean value of the problem Y in Eq. (5), and the second moment, σ^2_Y, represents the variance of Y in Eq. (6). The FOSM method equations are provided below where term (dg/dx) indicates the sensitivity of the response to the changes of variable x.

$$Y = g(x), \qquad x = \{x_1, x_2, ..., x_n\} \tag{3}$$

$$Y \approx g_0 + (dg/dx)_0 (x - x_0) \tag{4}$$

$$\mu_Y = E[g(x)]$$

$$\approx E[g_0 + (dg/dx)_0 (x - x_0)] \qquad \approx g_0 + (dg/dx)_0 (\mu_x - x_0) \tag{5}$$

$$\sigma^2_Y = E[g^2(x)] - \mu^2_Y$$

$$\approx E[g_0{}^2 + \{(dg/dx)_0 (x - x_0)\}^2 + 2g_0 (dg/dx)_0 (x - x_0)] - \mu^2_Y$$

$$\approx g_0{}^2 + (dg/dx)_0{}^2 \sigma^2_x + 2g_0 (dg/dx)_0 (\mu_x - x_0) - \mu^2_Y \tag{6}$$

In case of $x_0 = \mu_x$, Eq. (5) and Eq. (6) are written as Eq. (7) and Eq. (8), respectively. According to the Eq. (7) and Eq. (8), the mean and variance of the problem are determined based on the corresponding values of the input variable.

$$\mu_Y = g(\mu_x) \tag{7}$$

$$\sigma^2_Y \approx (dg/dx)_0{}^2 \sigma^2_x \tag{8}$$

MONTE CARLO SIMULATION ANALYSES RESULTS

As discussed in MC simulation technique description in the previous section, random soil samples were generated based on randomly varying soil input parameters. Then, soil responses were obtained by performing dynamic analysis for randomly varying soil samples with different characteristics. The considered input random soil parameter is G_0 since it was recognized as the most effective parameter in Tornado diagram analysis that mainly controls the soil dynamic behavior. Excluding soil shear modulus, the other soil properties were considered constant and given their BE value in the simulation of soil samples. The depth of 30 m was the only studied soil depth in this section since this depth is very common in the site classification consideration of regulations. The input seismic excitation was the one used in Tornado diagram analysis.

The interested response for studying in this part of research is PGA as an informative earthquake intensity measure parameter. Figure 5 displays results obtained from the MC simulation for soil medium of 30 m depth with varying G_0 values. According to Figure 5, the PGA values are sorted against G_0 quantities which show that by increasing of the G_0 the value of PGA at soil surface increases. Since soil G_0 is intensely related to the soil stiffness and strength, thus it could be concluded that stiffer soil experiences larger values of PGA. The frequency of the PGA occurrence as a bar chart is also depicted in Figure 5 which shows that PGA values were lognormally distributed with the mean and CV values equal to 0.384g and 0.128, respectively. On the other hand, PGA on average is amplified by 1.1 compared to the PGA of 0.35g at the bedrock.

Since MC simulation uses the results of a number of simulations in order to calculate the result, sufficient number of samples should be analyzed. The required number of samples was determined with the aid of a convergence test. In the convergence test the soil samples were increasingly simulated and analyzed and the cumulative mean and σ of the results were calculated at the end of each step of analyses. By dividing the cumulative mean and σ to the total value of mean and σ at each step a ratio was obtained. When the ratio tends to 1, the number of simulations is considered well enough and the results are assumed acceptable. Figure 5 also displays the convergence test outcome as a biaxial plot with one axis for the calculated ratio and the other for the number of analyzed samples. According to the results, 150 analyzed samples using MC meet the requirements of convergence and presumed reliable for the result calculation.

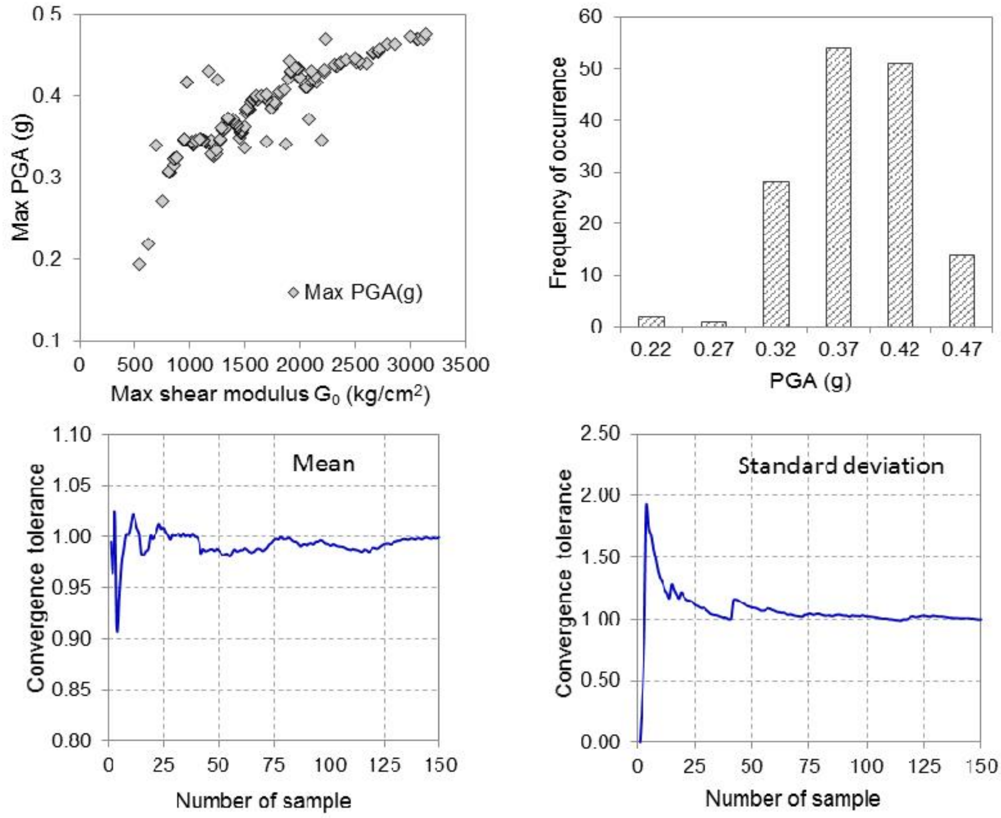

Figure 5. Probabilistic analysis results using MC simulation technique

RESULTS OF MONTE CARLO APPROACH AND FOSM IN COMPARISON

MC as a full probabilistic analytical approach is very time consuming. Despite the exact solution that MC approach provides, its application is not feasible in practical cases especially for problems which have a large number of degrees of freedom. On the other hand, FOSM as an approximate method in comparison to MC is very time and cost saving. It needs only a few number of simulations to solve the problem and calculate the response diversity. In this paper, to ensure the accuracy of FOSM method in predicting the probabilistic seismic response of the soil domain, its results were compared to the results of MC.

In order to perform the FOSM method analysis in this study, similar to what we did in MC simulations, the soil G_0 was the only considered variable parameter represented as x in FOSM formulations. The uncertainty of the input variable is entered in the simulations by defining three different values for G_0 including BE, LB and UB. Three soil samples each considering one of the three defined values of G_0 and depth of 30 m were generated. In the simulations, the other soil characteristic parameters were given their constant BE value. The applied earthquake record as seismic excitation was the one used in Tornado and MC analyses. Soil domain samples were analyzed subjected to this record and the seismic responses were obtained.

In the FOSM formulations, the mean response was obtained by analyzing the soil sample simulated based on G_0 BE value (Eq. (7)). The term (dg/dx) that represents the response sensitivity to G_0 variability, was calculated based on the samples simulated considering the limits, LB and UB, of G_0 (Eq. (1) and (2)). According to Table 1, the BE and CV of G_0 are equal to 1700 and 0.5, so the LB and UB limits of G_0 are equal to 850 and 2550 kg/cm^2, respectively. According to Eq. (8), the variance of the soil seismic response, σ^2_Y, is calculated based on the term (dg/dx) and the variance of soil G_0, σ^2_x. It's noteworthy that the variance is the square of the standard deviation. Standard deviation simply represents the response variability because it has the same dimension as the response.

As noted before, FOSM needs only three simulations and the mean and variance of results can be calculated based on the method's formulations. The calculation of the mean and σ of PGA is presented below, as an instance. The procedure is exactly the same for the other responses.

The value of PGA acquired from FOSM simulations considering BE, LB and UB values of soil G_0 are equal to 0.394g, 0.32g and 0.438g, respectively. Using Eq. (7) and (8), the mean and variance of PGA are calculated as follows:

$$\text{«}_{PGA} = 0.394g$$

$$(dg/dx) = (0.438 - 0.32)/(2*850) = 6.94e^{-5}$$

$$\pm_{PGA}{}^{2} = (6.94e^{-5})^2 * 850^2 = 0.003481 \qquad \pm_{PGA} = (0.003481)^{1/2} = 0.059g$$

Utilizing FOSM, the mean and σ of any seismic response of soil domain could be determined and compared to those obtained from MC simulation analyses. The results of the comparison are presented in Figure 6 and Table 2 displaying the good agreement between the results obtained from two methods.

The mean of MC simulation responses was obtained by averaging 150 simulations' results. According to Table 2, the mean and σ of PGA, AF and spectral responses of SDOF obtained from two approaches showed a good congruence. FOSM method, despite being an approximate approach, can be reasonably used instead of MC for seismic response evaluation of the considered soil domain.

Table 2. Mean and σ of soil responses using MC and FOSM

Response	Monte Carlo		FOSM	
	Mean	σ	Mean	σ
PGA (g)	0.384	0.049	0.394	0.059
AF_{max}	2.19	0.26	2.26	0.25
PSA_{max}(g)	1.61	0.31	1.57	0.4
PSV_{max}(cm/s)	126.34	22.83	118.97	21.66
PSD_{max}(cm)	24.99	7.16	24.23	6.71

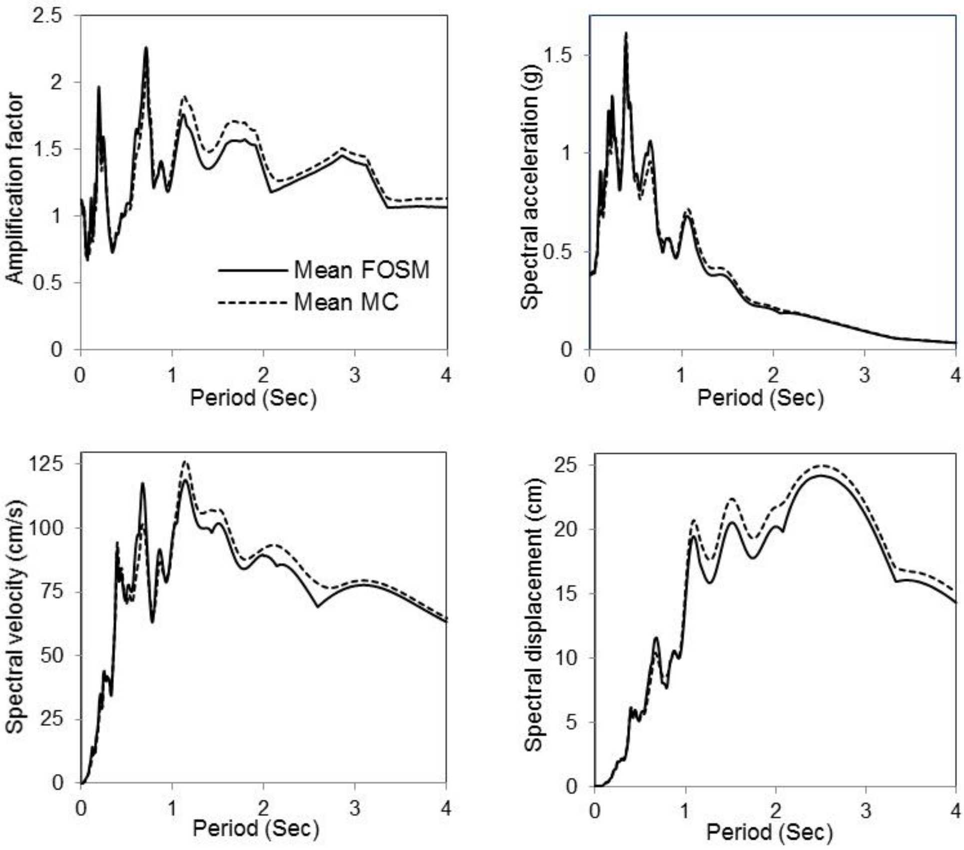

Figure 6. Comparison between the results of MC and FOSM

CONCLUSIONS

In this paper, the ground motions intensity considering soil properties uncertainty and depth to bedrock randomness was investigated. Depth randomness effect was studied by defining three different depths of soil domain. For this, three soil samples with different depths but similar mechanical and physical properties were simulated and analyzed subjected to 20 earthquake records. By averaging the result of 20 records, it was observed that by increasing the soil depth the maximum AF increased and occurred in longer periods which could be attributed to the longer oscillation period of the greater soil depth in which the resonance occurs. Furthermore, more resonant peaks in AF curve were observed at the greater depths of soil. As the soil depth became greater, the value of PGA decreased and its dispersion increased. Combining the results, it could be concluded that the soil depth severely influences the AF, PGA and the frequency content of ground motions that reach the surface, as well as the periods in which the maximum values occur.

Soil maximum shear modulus was recognized as the most effective soil parameter through Tornado diagram analysis. Two analytical approaches were employed for probabilistic assessments of the soil domain and the ground motions intensity. MC simulation technique was implemented by studying 150 soil samples of 30 m depth simulated considering randomly varying G_0 values. According to the results of MC, the PGA values were lognormally distributed with the mean value of 0.384g and CV of 0.128. PGA value increased by increasing the soil stiffness and strength as a result of the increase in G_0. It was also observed that PGA was equally affected by the soil G_0 uncertainty and record-to-record variability of the input seismic motions.

In order to save time and cost in probabilistic analyses, the efficiency of the FOSM approximate method was investigated. The comparison between the results of full probabilistic method of MC and approximate method of FOSM revealed the accuracy of FOSM as being a reasonable and time saving approach for evaluating the seismic response of nonlinear soil site with uncertain soil properties.

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