ESTIMATING MAXIMUM POUNDING FORCE BETWEEN TWO ADJACENT STRUCTURES

Sepideh RAHIMI
PhD Candidate in Earthquake Engineering, Department of Civil and Environmental Engineering, TarbiatModares University,Tehran, Iran
se.rahimi@modares.ac.ir

Masoud SOLTANI MOHAMMADI
Associate professor in Earthquake Engineering, Department of Civil and Environmental Engineering, TarbiatModares University, Tehran, Iran
msoltani@modares.ac.ir

Keywords: Structural Pounding, Pounding Force, Random Vibration, Statistical Relations, Bouc-Wen Model.

ABSTRACT

Seismic pounding between adjacent buildings with inadequate separation and different dynamic characteristics can cause severe damage to the colliding buildings. Therefore, the earthquake resistant design of structures due to pounding needs the knowledge of maximum pounding force. Efficient estimation of the maximum pounding force is required to control the extent of damage in adjacent structures. In this paper, an analytical approach on the basis of statistical relations is presented for the approximate computation of maximum pounding force between two adjacent single-degree-of-freedom (SDOF) systems subjected to stationary and non-stationary excitations. The pounding effect has been simulated by applying the nonlinear viscoelastic model. The proposed approach can significantly save computational costs by obviating the need for performing dynamic analysis and accurately estimating the maximum of pounding force. Accuracy of the proposed approach in comparison with the exact method is investigated using nonlinear dynamic analysis.

INTRODUCTION

Among different cases of structural damage, seismic pounding of the buildings with inadequate separation distance has been identified as the cause of severe damage in past earthquakes. Such poundings can be expected in major cities which have valuable under-construction lands and contain many buildings with small distances in between. Pounding between adjacent structures can produce large acceleration demands on the floors which are directly involved in collisions (Cole et al., 2010). So, efficient estimation of the maximum pounding force would help engineers in the seismic evaluation processes of closely-spaced structures to control the extent of damage.

Researchers have proposed numerous pounding models in order to model the poundings between buildings, such as linear elastic (Maison and Kasai, 1992), linear viscoelastic (Anagnostopoulos, 1992), nonlinear elastic (Chau and Wei, 2001) and nonlinear viscoelastic model (Jankowski, 2006). The linear elastic model consists of a linear spring that does not take the energy loss during contact. The linear viscoelastic model is a more precise model than the linear elastic model and considers the energy loss during collisions. Anagnostopoulos, (1992) used this model for simulating the pounding between adjacent SDOF systems in series. Maison and Kasai, (1992) simulated pounding between a light high-rise building and a massive low structure. In that study, a single linear spring was placed at the roof level of the lower structure for pounding force modeling procedure. The linear elastic model was also used by Anagnostopoulos and Spiliopoulos, (1992) on adjacent lumped mass models of 5 and 10-storey buildings with bilinear force deformation characteristics. For modeling of the pounding force more realistically, a nonlinear elastic model was employed by a number of researchers. Davis, (1992) used this model to simulate pounding of SDOF system against infinitely rigid adjacent system. Also, Jing and Young, (1991) as well as Chau and Wei, (2001) used nonlinear elastic model to analyze the pounding between two adjacent structures of different dynamic properties. But in these researches, the structural models were linear SDOF systems and furthermore the main limitation of nonlinear elastic model is that it cannot consider the energy dissipated during impact. But, the nonlinear viscoelastic model may be considered somewhat more accurate than the other mentioned ones. High efficiency of the nonlinear viscoelastic model in terms of simulating structural responses to the pounding effect has been indicated in the previous studies. Jankowski, (2006) used this model to determine pounding force response spectra for closely-spaced SDOF systems.

In spite of the fact that the research on pounding between adjacent systems has been recently much advanced, the studies didn’t propose a closed form solution for pounding force because the value of pounding force depends on different parameters such as damping ratio, mass of structures, gap distance and specially characteristics of the chosen ground motions which could change the value of pounding force. Therefore, the earthquake resistant design of structures due to pounding needs the knowledge of maximum of the pounding force with considering record-to-record uncertainties. For this purpose, nonlinear dynamic analysis is considered. However, this method is very time consuming with high computational cost, because many dynamic analyses should be done in applying the record-to-record uncertainties. Therefore, this paper aims at proposing a closed form algorithm to achieve maximum impact force between two adjacent systems with considering record-to-record uncertainties by using probabilistic methods. To propose a reliably exact comprehensive relation considering the maximum pounding force, several systems with Bouc–Wen (BW) hysteresis model (including a variety of hysteretic patterns) have been studied under input excitations with spectral density as a representative for a range of records in a particular area. Using the proposed relationship can obviate the essential need for performing dynamic analysis to evaluate the pounding phenomena and help in finding its more accurate characteristics in a less computational time. This relation is verified by nonlinear dynamic analysis for two adjacent nonlinear systems under records which are simulated for a specified spectral density as stationary and non-stationary excitations.

MODELLING OF SDOF SYSTEM

In this study, a single degree of freedom system is simulated by an oscillator with a classic Bouc–Wen model of hysteresis (Wen, 1967). One of the features of this model is the ability to consider different hysteresis models for a structure by changing including parameters. The model’s differential equation is as follows:

$$m\ddot{x}(t)+c\dot{x}(t)+\alpha kx(t)+(1-\alpha)kz(t)=P(t) \quad (1)$$

$$\dot{z}=A\dot{x}-\beta|\dot{x}||z|^{n-1}z-\gamma\dot{x}|z|^{n} \quad (2)$$

The first equation is the differential equation for the motion of a SDOF system which is a function of x and z, and z is a virtual displacement with differential equation as Eq. 2. In order to solve a problem, the mentioned equations haveto be solved simultaneously, so that by changing the existing parameters, e.g. γ and β, n, α and A different hysteresis models could be extracted (Fig. 1). The difference between these models and multi-linear ones presented by others like Clough and Takeda is that later models could not be expressed by one specific equation, while the Bouc–Wen model is a smooth model expressed by only one equation so that in these problems containing loading and unloading, the state of response would be determined easier.

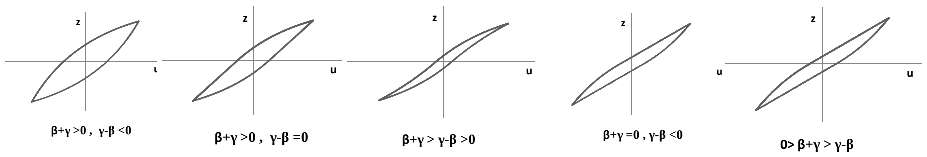

Figure 1. Classic Bouc-Wen models for different values of β, γ(Wen, 1967)

INPUT EXCITATION

Contact is a complex dynamic phenomenon that strongly depends on the input excitation. Therefore, interpreting the contact phenomenon could not be confined to a limited number of earthquakes and large number of records would be needed. But, this issue is somehow difficult to do, since the variety and numbers of the existing processed or recorded accelerations are low, and there is an initial need for making artificial acceleration time history. Which are produced by Tajimi-Kani approach is adopted herein (Tajimi, 1960). Tajimi-Kanai model is a filtered white power spectral density (G_0) that has been passed over a layer of soil with a natural frequency of (ω_g) and damping ratio of (ξ_g). The spectral density function of this model is:

$$S_g(\omega) = \frac{\omega_g^4 + 4\xi_g^2 \omega_g^2 \omega^2}{(\omega_g^2 - \omega^2)^2 + 4\xi_g^2 \omega_g^2 \omega^2} S_0 \tag{3}$$

Using the Elghadamsi's results, constant values for ξ_g, G_0 (cm^2/sec^3), ω_g(rad/sec), are respectively considered as 0.34, 94.76, 18.34 for alluvial terrain (Elghadamsi et al., 1988).

DISPLACEMENT VARIANCE UNDER STATIONARY RECORD

For determination of standard deviation, frequency response of the system hasto be detected by using of the system's equation of motion (Eq 1, 2) which is initially as partial derivative equations. Solving these equations is mainly nonlinear with no exact solution. In some cases, approximate solutions are derived by simple methods as linearization method.For this purpose, the governing differential equations of the BW model have to be converted into linear form. The parameters for the equivalent linear Classic BW are introduced by adopting the theory of minimizing least square error as follows (Wen, 1980). An equivalent linear equation will be looked for in the form:

$$\dot{z} = c_e\, \dot{x} + k_e\, z \tag{4}$$

The values of coefficient are:

$$c_e = A - \beta F_1 - \gamma F_2 \;,\; k_e = -\beta F_3 - \gamma F_4 \tag{5}$$

where functions F_i, i=1, 2, 3, 4 are given by:

$$F_1 = \frac{\sigma_z^n}{\pi}\Gamma\left(\frac{n+2}{2}\right)2^{n/2} I_s \;,\; F_2 = \frac{\sigma_z^n}{\sqrt{\pi}}\Gamma\left(\frac{n+1}{2}\right)2^{n/2},\; F_3 = \frac{n\sigma_{\dot{x}}\sigma_z^{n-1}}{\pi}\Gamma\left(\frac{n+2}{2}\right)2^{n/2}\times\left\{\frac{2(1-\rho_{\dot{x}z}^2)^{(n+1)/2}}{n} + \rho_{\dot{x}z} I_s\right\}$$

$$F_4 = \frac{n}{\sqrt{\pi}}\rho_{\dot{x}z}\sigma_{\dot{x}}\sigma_z^{n-1}\Gamma\left(\frac{n+1}{2}\right)2^{n/2} \;,\; I_s = 2\int_l^{\pi/2}\sin^n\theta\, d\theta \;,\; l = \tan^{-1}\left(\frac{\sqrt{1-\rho_{\dot{x}z}^2}}{\rho_{\dot{x}z}}\right) \tag{6}$$

$$\sigma_z = \sqrt{E[z^2]},\; \sigma_{\dot{x}} = \sqrt{E[\dot{x}^2]},\; \rho_{\dot{x}z} = \frac{E[\dot{x}z]}{\sqrt{E[\dot{x}^2]\, E[z^2]}}$$

Based on the linear coefficients, the frequency response function for linear BW is:

$$|H(\omega)|^2 = \frac{1}{\left(-\omega^2 + \alpha\omega_0^2 + \frac{(1-\alpha)\omega_0^2\omega^2 c_e}{\omega^2 + k_e^2}\right)^2 + \left(2\xi\omega_0\omega - \frac{(1-\alpha)\omega_0^2\omega c_e k_e}{\omega^2 + k_e^2}\right)^2} \tag{7}$$

Standard deviation of system's response would be calculated by Eq. (8):

$$\sigma_u^{\,2} = E[u^2] = \int_{-\infty}^{+\infty} S_{yy}(\omega)\, d\omega = \int_{-\infty}^{+\infty} |H(\omega)|^2\; S_g(\omega)\, d\omega \tag{8}$$

In this equation, standard deviation of displacement depends on the frequency response function $H(\omega)$ and input excitation density function $S_g(\omega)$ (Eq.3). In this research, to obtain statistical information of response, a large number of dynamic analyses were performed. Using sensitivity analysis, statistical properties of the analyzed responses from 200, 300 and 400 records have the appropriate converge.The convergence criterion is the standard deviation of the response obtained from the records. A difference in the standard deviation of between 300 records and 200 records is equal to 1.5 percent and between 400 records and 300 records is equal to 0.6 percent. Due to the small difference, in this study, 300 records were used to determine the statistical information of exact results.

DISPLACEMENT VARIANCE UNDER NON-STATIONARY RECORD

A general approach is to model the non-stationary excitation as:

$$F(t) = A(t)\ W(t) \tag{9}$$

where*A(t)* is a deterministic, modulating function and *W(t)* is a stationary random process. The letter is usually taken to be white noise, or filtered white noise. When a modulating function is introduced in connection to the Tajimi-Kanai filter, non-stationary model of seismic action is produced. The modulating function employed in this case is that of Shinozuka and Sato, (1967) given by:

$$A(t) = \frac{1}{C}(e^{-B_1 t} - e^{-B_2 t}) \quad , \quad C = \max(e^{-B_1 t} - e^{-B_2 t}) \tag{10}$$

The parameters $B_1=0.085$ and $B_2=0.17$ have been used, which correspond to a long duration earthquake. The variance of non-stationary excitation can be found by following numerical approximations developed by Michaelov et al. (2001), for the simple oscillator with natural frequency μ_0 and damping ratio $\P$ and the excitation modulation given in (11):

$$\pm_X^2(t) \approx \pm_{X\infty}^2 C^2 \{\frac{\P}{\P_1}\left[\exp(-2B_1 t) - \exp(-2\P\mu_0 t)\right] + \frac{\P}{\P_2}\left[\exp(-2B_2 t) - \exp(-2\P\mu_0 t)\right]$$
$$-2\frac{\P}{\P_m}\left[\exp(-2B_m t) - \exp(-2\P\mu_0 t)\right]\} \tag{11}$$

$$\P_1 = \P - \frac{B_1}{\mu_0} \ , \ \P_2 = \P - \frac{B_2}{\mu_0} \ , \P_m = \frac{\P_1 + \P_2}{2} \ , \ B_m = \frac{B_1 + B_2}{2} \tag{12}$$

And $\pm_{X\infty}^2$ denotes the variance of the stationary response of the oscillator to stationary filtered white noise. To avoid this computational hurdle, the authors considered approximations of the above function as:

$$\pm_X^2(t) \approx A^2(t)\, \pm_{X\infty}^2 \tag{13}$$

The approximate procedure can significantly facilitate the utilization of non-stationary models in engineering practice, since it avoids computational difficulties. The method is based on the approximation of a non-stationary process by an ‘‘equivalent’’ stationary process. The variance of this ‘‘equivalent’’ stationary process is$\pm_{eq}^2$, and the process must be considered for the time interval *[0, Teq]* in order to create the same probability as the original non-stationary process *X(t).* The formulas of the equivalent variance and the equivalent duration of the non-stationary process developed by Michaelov et al., (2001) are:

$$\pm_{eq}^2(n_1) = \frac{I(n_1+1)}{I(n_1)} \ , \quad T_{eq}(n_1) = \frac{I(n_1)}{\pm_{eq}^{2n}(n_1)} \ , \ I(n_1) = \int_0^T \pm_X^{2n_1}(t)\, dt \tag{14}$$

CONTACT BETWEEN TWO ADJACENT SYSTEM

Herein, the seismic pounding force between two adjacent SDOF systems, as shown in Fig. 2, is presented. It is worth noting that the left and right structures are called structures 1 and 2, respectively. Dynamic equation of motion which includes pounding force during impacts can be written as follows:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{Bmatrix}+\begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{Bmatrix}+\begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix}+\begin{Bmatrix} F(t) \\ -F(t) \end{Bmatrix}=-\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_{g1}(t) \\ \ddot{x}_{g2}(t) \end{Bmatrix} \quad (15)$$

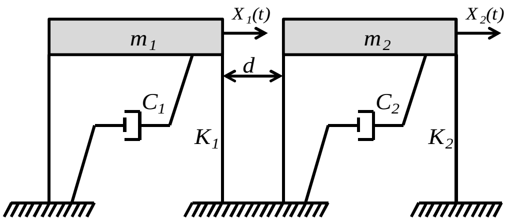

Figure 2. Model of interacting structure

In this equation, $x_i(t), \dot{x}_i(t), \ddot{x}_i(t), C_i$ and K_i are the values of horizontal displacement, velocity, acceleration, damping coefficient and stiffness coefficient for the structures 1, 2 respectively (i=1, 2). Moreover, $\ddot{x}_{gi}(t)$ denotes the acceleration input ground motion and $F(t)$ accounts for the value of pounding force. In order to solve the above equation numerically, the value of pounding force should be determined using an appropriate pounding model. A nonlinear viscoelastic model is used for simulating the interacting structure by the following formula:

$$\begin{cases} F(t)=0 & for \quad ¢(t)\le 0 \\ \quad (no\,contact) & \\ F(t)=\bar{\ }¢^{\frac{3}{2}}(t)+\bar{c}\dot{¢}(t) & for \quad ¢(t)>0 \quad and \quad \dot{¢}(t)>0 \\ \quad (contact-approach\ period) & \\ F(t)=\bar{\ }¢^{\frac{3}{2}}(t) & for \quad ¢(t)>0 \quad and \quad \dot{¢}(t)\le 0 \\ \quad (contact-restitution\ period) & \end{cases} \quad (16)$$

$$\begin{cases} ¢(t)=x_1(t)-x_2(t)-d \\ \dot{¢}(t)=\dot{x}_1(t)-\dot{x}_2(t) \end{cases}$$

where $\bar{\ }$ is the impact stiffness parameter that depends on the material properties of the pounding structure and the geometry of the contact area and $\bar{c}(t)$ is the impact element's damping, which at any instant of time can be obtained from the formula:

$$\bar{c}(t)=2¶_h\sqrt{\bar{\ }\sqrt{¢(t)}\left(\frac{m_1m_2}{m_1+m_2}\right)},\ ¶_h=\frac{9\sqrt{5}}{2}(1-e^2)\frac{(1-e^2)}{e(e(9®-16)+16)} \quad (17)$$

where the coefficient of restitution, e, is defined as the ratio of separation relative velocities of the bodies after impact, $\dot{¢}_f$, to their approaching relative velocities before impact ($\dot{¢}_0$):

$$e=\frac{|\dot{¢}_f|}{\dot{¢}_0} \quad (18)$$

Based on the energy laws, maximum deformation isequal to:

$$¢_{max}=\left(\frac{5}{4}\frac{m_1m_2(\dot{¢}_f)^2}{(m_1+m_2)\bar{\ }}\right)^{\frac{2}{5}} \quad (19)$$

Asthe maximum deformation ($¢_{max}$) only depends on the relative velocity after pounding ($\dot{¢}_f$) and according to Eq. 18, the velocity after pounding is a coefficient of relative velocity before pounding; therefore, by only knowing the expected extreme value of relative velocity before the pounding, the expected extreme value of pounding force is equal to:

$$F_e = -\dot{¢}_e^{\frac{3}{2}} = -\left(\frac{5}{4}\frac{m_1 m_2 (e\dot{¢}_e)^2}{(m_1+m_2)}-\right)^{\frac{3}{5}} \tag{20}$$

where $\dot{¢}_e$ is the expected extreme value of relative velocity before pounding and F_e is the expected extreme value of pounding force.

EXPECTED EXTEREME VALUE OF RELATIVE VELOCITY

Adjacent structural systems "1" and "2" are modelled as SDOF systems, the seismic excitation is a Gaussian, zero mean excitations and the relative velocity response process $v_{rel}(t)$ is given by:

$$V_{rel}(t) = V_1(t) - V_2(t) \tag{21}$$

The mean square response of Eq. 21 is equal to:

$$\begin{aligned} \pm^2_{V_{REL}} &= E\left\{V^2_{REL}(t)\right\} = E\left\{[V_1(t)-V_2(t)]^2\right\} = E\left\{V_1^2(t)\right\} + E\left\{V_2^2(t)\right\} - 2E\left\{V_1(t)V_2(t)\right\} \\ &= \pm^2_{V_1} + \pm^2_{V_2} - 2E\left\{V_1(t)V_2(t)\right\} \end{aligned} \tag{22}$$

where $E\{\}$ is the expected value and $\pm^2_{V_1}, \pm^2_{V_2}$ are the standard deviation of displacement of systems 1 and 2. Thestandard deviation of the velocity of the system "*i*" is equal to:

$$\pm_{V_i}^2 = E[V_i^2] = \int_{-\infty}^{+\infty} µ^2 S_{uu}(µ)\,dµ = \int_{-\infty}^{+\infty} µ^2 |H_i(µ)|^2 S_g(µ)\,dµ \tag{23}$$

and the expected value of velocity of two systems is equal to:

$$E\{V_A(t)V_B(t)\} = \int_{-\infty}^{+\infty} µ^2 H_A(µ) S_g(µ) H_B(µ)^*\,dµ \tag{24}$$

For a zero-mean stationary Gaussian process $V_{rel}(t)$, Davenport has shown the mean of extreme-values of relative velocity are given by the approximate relation:

$$\dot{¢}_e = \left((2\ln(³T))^{0.5} + \frac{¥}{(2\ln(³T))^{0.5}}\right)\pm_{V\,rel} \tag{25}$$

$$³ = \frac{\pm_{V_{rel}}}{® \pm_{X_{rel}}}, \quad ¥ = 0.5772, \quad T = time \quad duration \tag{26}$$

For a zero-mean non-stationary process, the mean of extreme values of velocity for non-stationary processes is equal to:

$$\dot{¢}_e = \left((2\ln(³T_{eq}))^{0.5} + \frac{¥}{(2\ln(³T_{eq}))^{0.5}}\right)\pm_{eq\,V\,rel} \tag{27}$$

In order to determine the maximum pounding force, two adjacent SDOF systems with the BW model and different periods (T_1=1.2 sec, T_2=0.4, 0.6, 0.8 sec) were analyzed under 300 stationary and non-stationary records with Tajimi-Kanai spectral density and then, the average value of the maximum pounding force of these records were obtained. Also, the maximum of pounding force was investigated on the basis of statistical relations and the proposed closed-form method without any need to perform exact dynamic analysis.Figure. 3 shows different steps to find out the value of maximum pounding force from the exact solution and the proposed method. Comparisons between expected extreme values of pounding forces obtained from the performed analyses by use of 300 stationary and non-stationary records and from the proposed algorithm are presented in Tables 1 and 2.

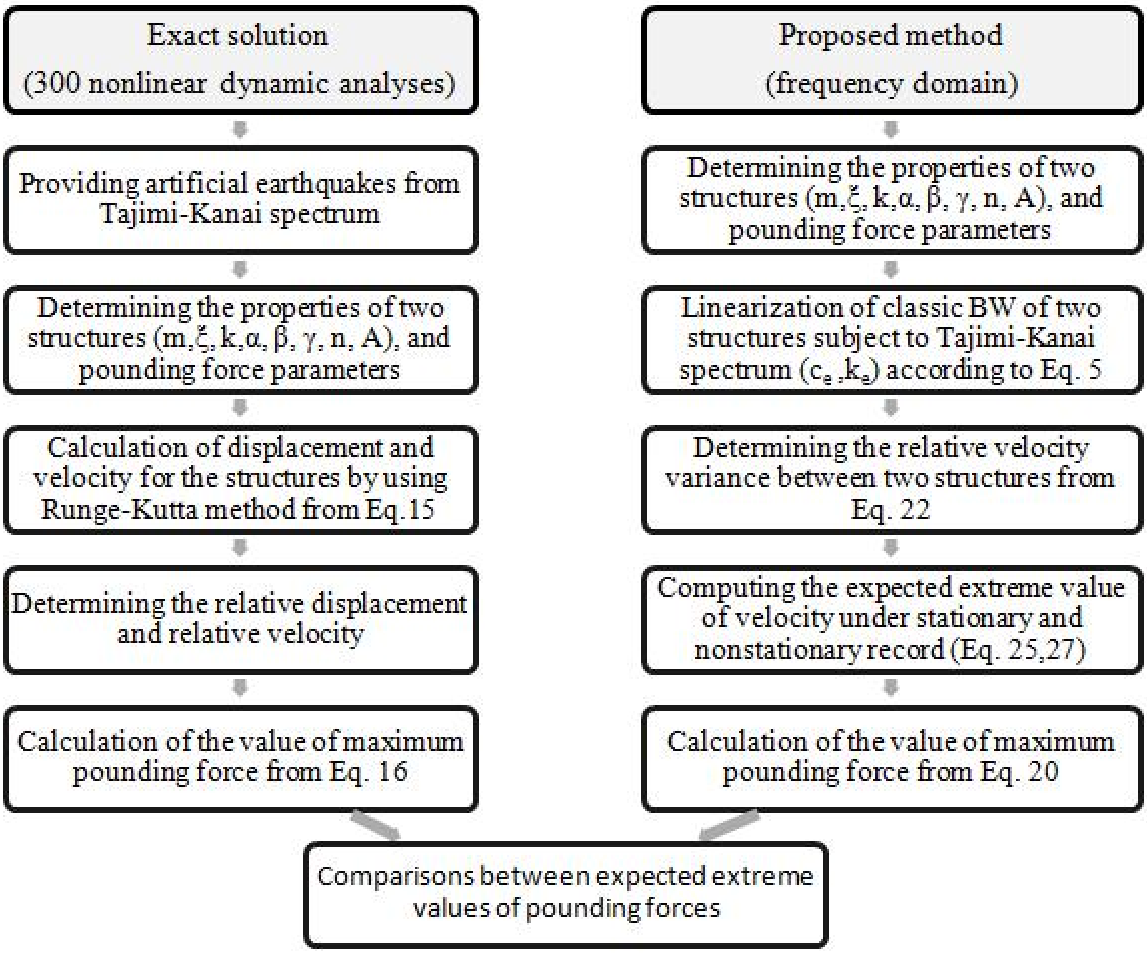

Figure 3.different steps to find out the value of maximum pounding force from exact and proposed method

Table 1. Expected extreme value of pounding force between SDOF systems under stationary records

No.	T_1, T_2 (sec)	300 nonlinear dynamic analyses (exact method),(kg)	Frequency domain (proposed method) ,(kg)
1	1.2, 0.4	48873	49249
2	1.2, 0.6	44496	43563
3	1.2, 0.8	37979	36573

Table 2. Expected extreme value ofpounding force between SDOF systems under non-stationary records

No.	T_1, T_2 (sec)	300 nonlinear dynamic analyses (exact method) ,(kg)	Frequency domain (proposed method) ,(kg)
1	1.2, 0.4	34888	36968
2	1.2, 0.6	31599	31767
3	1.2, 0.8	27639	27329

According to Table 1 and 2, the proposed algorithm has the capability of estimating the expected extreme value of the pounding force of two adjacent systems under stationary and non-stationary excitation.

CONCLUSIONS

The expected extreme value of pounding force between two single degree of freedom systems subject to stationary and non-stationary excitation was investigated on the basis of statistical relations and a closed-form method was presented to approximately detect the expected extreme value of pounding force without any need for performing exact dynamic analysis. Adjacent buildings with similar or different BW hysteretic behaviours can be easily modeled using the proposed approach. Compared to the exact dynamic analysis procedure, the proposed approach had acceptable probabilistic results in analytical terms for the expected extreme value of the pounding force of SDOF systems under stationary and non-stationary Gaussian excitations.

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