# CALCULATION OF LATERAL LOADS ON BUILDING BASED ON ENERGY METHOD

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## ABSTRACT

Because of the facilities of the static analyses, the designers have an interest to apply them in their design purposes. Therefore, most of the constructional codes are intended to use equivalent static loads (ESLs) rather than dynamic loads.Additionally, they use dynamic factors to transform dynamic loads into equivalent static loads. Many researches had studied about different methods to find the most rational equivalent static loads. Generally, there is two different methods. Exact and approximated ways had been proposed to achieve ESLs. Approximated methods are divided into displacement-based approach and stress-based approach according to the constraints of the problem. Here, an approximated method is derived based on a new approach. It will propose the energy dissipated in the structure to find out ESLs.The method is proposed for transformation of seismic loads (in the form of standard response spectra) into equivalent dynamic loads based on the finite-element method. By using the implemented code in the MATLAB environment and then solving an optimization sub-problem (sequential quadratic programming), transformation of seismic loads into equivalent static loads is available.

## INTRODUCTION

Most of the forces in practical applicationsare dynamic and variable in the time domain. A dynamic analysis of structures requires the large amounts of information processing and data interpretation. Accordingly, static loads could be utilized as a substitute for earthquake loadings, if they produce the same responses as the dynamic loads do. Because of the facilities of the static analyses, the users of building codes have an interest to apply them in their design purposes. How to transform a dynamic load into a proper static load is becoming more and more important for the good evaluation and the definition of the applied load in design procedure. The transformation of dynamic loads into equivalent static loads has been studied in civil engineering for analysis and design under earthquake loads (Cheng and Juang, 1988; Truman and Cheng, 1990; Cheng and Truman, 1983; Truman and Petruska, 1991; Cheng and Chang, 1988; Austin et al., 1987). To find a unique solution, presumptions are needed, such as the shape or the location of the load. To meet the quality and to overcome the absence of a unique solution, an equivalent static load (ESL) is proposed (Choi et al., 1996; Choi and Park, 1999a,b). Two different methods are proposed for ESLs. The analytical method for an exact ESL generates identical response fields with those from dynamic loads.The approximated ESLs are calculated in minimum that satisfies the constraints (Akbari and Sadoughi,2012). As a constraint, the concept of energy seems to best explain the structural response to a strong ground motion. The earthquake effect on the structure can be considered as energy input, a function of the structural properties and characteristics of the earthquake ground motion. Therefore, seismic design becomes the balance of the energy demand and capacity of the structure. Starting from the early work of Housner (1956), there exists

research on the energy-based concepts for structural systems (Zahrah and Hall, 1984; Uang and Bertero, 1990; Fajfar et al., 1992; Ye and Otani, 1999; Riddell and Garcia, 2001). Previous studies have been mainly focused on single degree of freedom (SDOF) systems and concluded that the energy input is much more sensitive to the ground motion parameters than the structural properties (such as the period, ductility and strength), particularly in the medium and long period structures. Empirical formulas have been proposed to estimate the energy input for the SDOF system mainly based on ground motion characteristics. Uang and Bertero (1990) derived relative and absolute energy equations in their study.
According to the Uang and Bertero (1990) research, relative input energy of an SDOF system can provide a good estimate of the input energy for multi-story buildings. Uang and Bertero (1990) conducted a shaking table test of 6 story steel frame structure subjected to a specific ground motion record and the correlation between experimentally measured energy equivalent velocity for the multi storey structure and the calculated energy equivalent velocity for an SDOF system was found to be very good. Similarly, Tso et al (1993) stated that there is a good correlation of input energy between low rise ductile moment resisting frames and equivalent SDOF systems. However, for high rise frames, the contributions of higher modal responses become significant and the use of equivalent SDOF systems may underestimate the energy demand on the building.

## Finite Element modeling

Gravity andseismic earthquake loadings have been included in the finite element model. In civil engineering, linear design of structures under earthquake loading is a common methodology. For that reason, linear behavior of the structure is taken into account here. Two dimensional frame elements have been considered in this study. Every elements and their DOFs are represented (See Fig 1). For numerical calculation of responses, the required vectors and matrices for the elements are calculated using standard finite element analysis (R.D. Cook et al 2002)

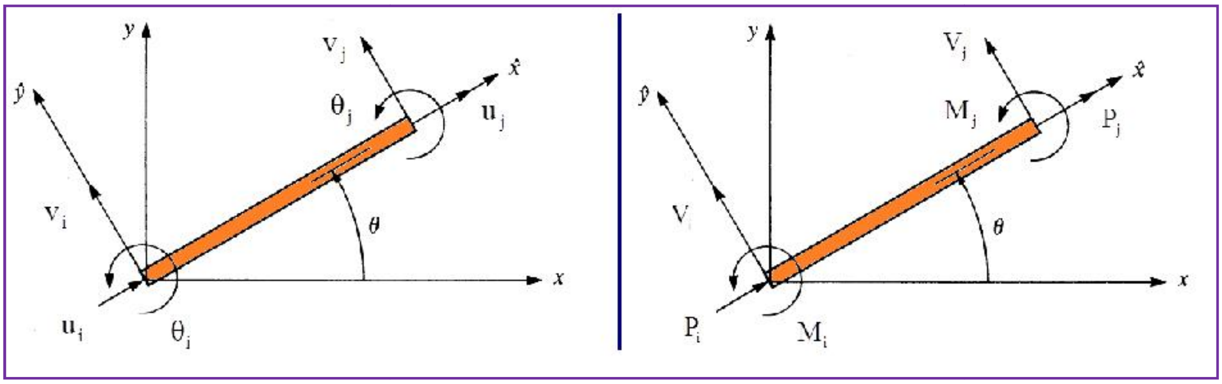


Figure 1. Frame elements with related DOFs (left) and corresponding internal forces (right)

## Proposed Methodology

### Dynamic response formulation

The ESLs are static loads that produce the same energy in a structure as the earthquake loadings.The process of ESLs calculation is presented in this section. The dynamic equation of motion for structures using the finite element method is explained as Eq. (1)

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = -\mathbf{M}\mathbf{r}\ddot{\mathrm{U}}_g \tag{1}$$

Where $\mathbf{M},\mathbf{C},\mathbf{K}$ are the mass, damping and stiffness matrices, respectively. Vectors of displacements, velocities and accelerations of a structure are defined by $\mathbf{U},\dot{\mathbf{U}},\ddot{\mathbf{U}}$, respectively. $\ddot{\mathrm{U}}_g$ is the ground acceleration, and $\mathbf{r}$ is the influence vector that shows the direction of the applied loading on the structure. This equation is a coupled one and would be an uncoupled one due to a diagonal damping matrix. Here, the modal approach ( $\mathbf{U} = \mathbf{\Phi Y}$ ) is applied for transformation of Eq.(1) into uncoupled second –order equation as Eq.(2).

$$\ddot{y}_n + 2\xi_n\omega_n\dot{y}_n + \omega_n^2 y_n = \frac{\mathbf{\Phi}_n^T}{M_n}\mathbf{Mr}\ddot{U}_g \tag{2}$$

Where, $\mathbf{\Phi} = [\mathbf{\Phi}_1, \mathbf{\Phi}_2, ....\mathbf{\Phi}_n]$ shows the modal matrix, $\mathbf{\Phi}_n^T = [¤_{11}, ¤_{12}, ...¤_{1n}]$ is the vector of normalized components for n-th mode shape, and $\mathbf{Y} = [y_1 y_2, ...y_{nm}]$ refers to the vector of modal displacements. $nm$ is the number of applied modes in the calculation of dynamic responses. $y_n, \dot{y}_n, \ddot{y}_n$, refer to the modal displacement, velocity and acceleration at n-th mode, respectively. $\omega_n$ is the frequency of n-th mode. Damping ratio and the modal mass of the n-th mode are defined by $¶_n$, $M_n$ is modal mass and is defined as Eq.(3)

$$M_n = \mathbf{\Phi}_n^T\mathbf{M}\mathbf{\Phi}_n \tag{3}$$

Response spectrum analysis has been applied to calculation of maximum values of responses at each mode.The maximum displacements and velocities of nodal vector is at each mode is computed as Eq.(4)

$$\mathbf{U}_{max}^n = \mathbf{\Phi}_n \frac{L_n}{M_n} S_d(\zeta, T_n) \tag{4}$$

$$\dot{\mathbf{U}}_{max}^n = \mathbf{\Phi}_n \frac{L_n}{M_n} S_v(\zeta, T_n)$$

Here, $S_d(\zeta, T_n), S_v(\zeta, T_n)$, refer to the modal displacement and velocity spectra respectively, and $L_n$ is the modal excitation factor and calculated as Eq.(5)

$$L_n = \mathbf{\Phi}_n^T\mathbf{Mr} \tag{5}$$

In this case, the complete quadratic combination (CQC) rule for the modal combination of peak values of each mode has been utilized

**Energy Based Calculation of ESLs**

Quantification of input energy is the first step in energy based approach. Before identifying the sources of energy dissipation, it must be clear how much energy is input to structure during an earthquake.Various types of energies dissipated by the structure during an earthquake are represented as Eq.(6).

$$E_K = \frac{1}{2}\dot{\mathbf{u}}^T\mathbf{M}\dot{\mathbf{u}} \tag{6}$$

$$E_D = \frac{1}{2}\dot{\mathbf{u}}^T\mathbf{C}\mathbf{u}$$

$$E_S = \frac{1}{2}\mathbf{u}^T\mathbf{K}\mathbf{u}$$

$$E_I = -(\mathbf{u}^T\mathbf{Mr}\ddot{U}_g)$$

In above formulas, $E_K$ is the kinetic energy of system, $E_D$ is the amount of energy dissipated by damping, $E_S$ is strain energy of structure and $E_I$ is the total input energy of earthquake. To relate dynamic loading to static ones, the energy of structure under specified applied static loading $E_{static}$, is represented by Eq.(7)

$$E_{static} = \frac{1}{2}\mathbf{u}^T\mathbf{K}\mathbf{u} \tag{7}$$

In linear elasticstatic analysis the following equation is used.

$$\mathbf{Ku} = \mathbf{P} \tag{8}$$

That $\mathbf{P}$, is the static load vector. Hence, we can rewrite Eq.(7) as the following form.

$$E_{static} = \frac{1}{2}\mathbf{U}_{Dy}^{T}\mathbf{P} \tag{9}$$

In above expression, $\mathbf{U}_{Dy}^{T}$ is the displacement vector under dynamic loading. Now, based on proposed criteria for achieving ESLs, the energy values should satisfy the following inequality:

$$E_K + E_D + E_S \leq E_{static} \tag{10}$$

And by substituting the statement of each energy type:

$$\frac{1}{2}\dot{\mathbf{u}}^{T}\mathbf{M}\dot{\mathbf{u}} + \frac{1}{2}\dot{\mathbf{u}}^{T}\mathbf{C}\mathbf{u} + \frac{1}{2}\mathbf{u}^{T}\mathbf{K}\mathbf{u} \leq \frac{1}{2}\mathbf{U}_{Dy}^{T}\mathbf{P} \tag{11}$$

Where $\mathbf{u}$ and $\dot{\mathbf{u}}$ are displacement and velocity responses derived by RSA procedure. Here we are facing an optimization problem to finding the $\mathbf{P}$ values. The suggested optimization algorithm is sequential quadratic programming (SQP) problem. Such problem is formulized as Eq.(12) in the computer code:

$$\begin{aligned} &\text{Minimize } \sum P_j^2 \\ &\text{subject to} \\ &E_K + E_D + E_S \leq \frac{1}{2}\sum_{i=1}^{n} u_j P_j \end{aligned} \tag{12}$$

Theanswer could be derived by calling an optimization toolbox in a computer program. Hence, the objective function is the square sum of matrices of $\mathbf{P}$ vector. With a linear inequality constraint described in Eq.(12) , the optimized values of $\mathbf{P}$ vector is achieved.

## Numerical Studies

To shed more light on the ESLs in this approach, thelateral loadings ontwo different loading systems(steel moment frame and braced frame) for six-story and twelve-story 2Dframes have been computed. For this purpose, the results of lateral loads on each story have been calculated using three:quasi-static, response spectrum and energy based methods. Similar simplifying assumptions have been applied on every case in analyzing procedure including: all sections of all elements are IPE300 profile,there is a 10 kN/m distributed loading on each beam, damping ratio is 0.05, the soil type is II,and the response modification factor is 1 such as the importance factor. For all cases, the value of A=0.35 has been taken into account. In the all frames, every bayshave four meters wide and each story has three meters height.Empirical periods of frames, for six-story braced and unbraced frames are 0.437 and 0.699 second,respectively. Empirical first periods of vibration for twelve- story braced, and unbraced frames are 0.735 and 1.176 second.The numerical periodsofprincipal mode for unbraced and braced six-story, three-bays frame are 0.897 and 0.415 second, respectively. İn addition,The numerical periodsofmajor mode for unbraced and braced twelve-story, three-bays frame are 1.848 and 1.786 second. Therefore, for quasi-static analysis, the selected period is obtiaed is follows:

$$T_{real} = \min\{T_{num}, 1.25T_{exp}\} \tag{13}$$

In the Table 1, the lateral forces for six-story, three-bay, unbraced and braced frames have been presented. These case arerelatively short to medium period structures, and first mode is usually dominent.The forces of code based methodare more than response spectrum method ones. İt is clear that quasi-static method is overestimate in compare with dynamic methods.For quasi static method, the Bfactor are equal to 1.63 and 2.50 for unbraced and braced frames, respectively. Therefore, the braced frame tolerated higher base shear in compare with unbraced one. Energy based method has been estimated the loads more largely than other methods for each frame. The reason is that the building has more displacement during earthquake and could dissipate more energy.

Table 1.Lateral loadings for a six- story, three-bay

| ESLs(kN) | **Un-braced frame (T=0.874 sec), weight=775 kN** | | | **Braced frame (T=0.415 sec), weight=776 kN** | | |
|---|---|---|---|---|---|---|
| | Quasi-static method | Response spectrum | Energy method | Quasi-static method | Response spectrum | Energy method |
| P1 | 21.0 | 18.1 | 34.9 | 20.2 | 32.3 | 23.6 |
| P2 | 42.1 | 36.2 | 85.1 | 63.5 | 64.7 | 47.1 |
| P3 | 63.1 | 54.3 | 131.7 | 123 | 97.0 | 70.7 |
| P4 | 84.2 | 72.4 | 170.4 | 192.7 | 129.3 | 94.3 |
| P5 | 105.2 | 90.5 | 198.7 | 267.5 | 161.7 | 117.9 |
| P6 | 126.3 | 108.6 | 215.3 | 343 | 194.0 | 141.4 |
| **Vb** | **442** | **375** | **830** | **679** | **495** | **1010** |

In the Table 2, the results of lateral loads for twelve-story, three-bay, unbraced and braced frames have been presented. These case are relatively large-period structures, and first mode is not usually dominent. The results of code based are more than response spectrum method ones. İt is clear that quasi-static method is overestimate in compare with dynamic methods. For quasi static method, the reflection factors (B) are equal to 1.25and 1.40 for unbraced and braced frames, respectively. Therefore, the braced frame tolerated higher base shear in compare with unbraced one. Energy based method has been estimated the loads more largely than other method for each frame. The reason is that the building has more displacement during earthquake and could dissipate more energy.

Table 2.ESLs for a twelve- story, three-bay

| **ESLs(kN)** | **Un-braced frame (T=1.848 sec), weight= 1552 kN** | | | **Braced frame (T=1.786 sec), weight= 1600kN** | | |
|---|---|---|---|---|---|---|
| | **Quasi-static method** | **Response spectrum** | **Energy method** | **Quasi-static method** | **Response spectrum** | **Energy method** |
| P1 | 8.7 | 6.2 | 10.6 | 9.8 | 6.4 | 2.3 |
| P2 | 17.4 | 12.3 | 26.4 | 19.5 | 12.8 | 7.9 |
| P3 | 26.1 | 18.5 | 42.8 | 29.3 | 19.2 | 16.4 |
| P4 | 34.8 | 24.6 | 58 | 39.1 | 25.6 | 27.3 |
| P5 | 43.5 | 30.8 | 72.8 | 48.8 | 32.1 | 40.2 |
| P6 | 52.2 | 36.9 | 86.6 | 58.6 | 38.5 | 54.8 |
| P7 | 60.9 | 43.1 | 99.2 | 68.4 | 44.9 | 70.6 |
| P8 | 69.6 | 49.2 | 110.5 | 78.2 | 51.3 | 87.4 |
| P9 | 78.3 | 55.4 | 120.3 | 87.9 | 57.7 | 104.9 |
| P10 | 87.1 | 61.5 | 128.5 | 97.7 | 64.1 | 122.8 |
| P11 | 95.8 | 67.7 | 134.9 | 107.5 | 70.5 | 140.9 |
| P12 | 104.5 | 73.8 | 139.3 | 117.2 | 76.9 | 159.1 |
| **Vb** | **679** | **480** | **1030** | **762** | **500** | **835** |

## Conclusion

This study applys a methodology to calculate the ESLs for skelton frames based on elastic energy method. Earthquake loading (acceleration spectra) is transformed into the equivalent static loads (ESLs). The numerical examples six-story and tweleve –story frame have been invistigated. The results of lateral loading using three distinct, quasi-static, response sepctrum and energy based optimization methods have been calculated and compared. Based on this study, following conclusions have been drawn:

1- Quasi-static method usually is over-estimate in compare with other methods. However, it uses only first mode charactrisitc of structures

2- For short period structures, the energy method is suitable and could dissapted more enetgy

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