PROBABILITY OF FAILURE MODE IN RC COLUMNSUSING BAYESIAN METHOD

Azad Yazdani
Associate Professor, University of Kurdistan, Sanandaj, Iran
a.yazdani@uok.ac.ir

KowsarYAZDANNEJAD
PhD Student, University of Kurdistan, Sanandaj, Iran
yazdannejadk@yahoo.com

Keywords: Bayesian Parameter Estimation, Uncertainty, Reinforced Concrete Column, Failure Mode

ABSTRACT

In order to help engineers better understand the behavior of concrete columns and to select an appropriate model for estimating the column capacity, a column classification method is required in engineering practice. Moreover, the information of the expected column failure mode (or column response) is helpful for engineers involved in the seismic assessment and retrofit of reinforced concrete buildings. This paper provides an approach to construct a probabilistic failure mode for concrete columns. The onset of column failure is defined as 20% loss in the lateral strength and three column failure modes (flexure failure, shear failure and flexure-shear failure) are considered. The database that consists of tests of reinforced concrete columns are divided into three categories in terms of span to depth ratio.The methodology developed by Gardoni et al. (2002) is adopted to construct the probabilistic capacity models for reinforced concrete columns.

INTRODUCTION

In current structural engineering practice, most drift capacity models are deterministicand do not explicitly account for all the prevailing uncertainties. For example, Elwoodand Moehle (2005) proposed a deterministic drift capacity model at shear failure basedon a database of 50 laboratory tests on reinforced concrete columns. This model onlyprovides a point prediction of the drift at shear failure and ignores the variability in thetest results. Approximately fifty percent of the columns included in the database experienced shear failure at drifts less than those estimated by the model. The deterministic model coefficients cannot reflect the epistemic uncertainties in the model(e.g., finite number of observations). In addition, the model error due to the model imperfection is not represented. Hence, it is needed to develop a probabilistic drift capacity model that defines not only a point prediction but also the variance of the model prediction.The main objective for this study is to help engineers better understand the behavior ofreinforced concrete columns through an appropriate column classification approach.

CAPACITYMODEL FORMULATION

In this study, the methodology developed by Gardoni et al. (2002) is adopted to constructthe probabilistic capacity models for reinforced concrete columns. This methodology iscapable of incorporating a wide range of information, including existing deterministicmodels, laboratory test data, field observations, and engineering judgment.According to this methodology, a probabilistic capacity model can bewritten in the following form(Gardoniet al., 2002):

$$C(x,\theta,\sigma) = \hat{C}(x) + \gamma(x,\theta) + \sigma.\varepsilon \quad (1)$$

where $\hat{C}(x)$denotes the selected deterministic capacity model; x denotes a set of basiccapacity and demand variables (e.g., material properties, member dimensions, appliedloads); $\theta = (\theta_1, \theta_2, \dots \theta_k)$ represents a set of *k* unknown model coefficients; $\gamma(x,\theta)$denotes the model correction term; $\varepsilon$is a normal random variable with zero mean andunit variance; and σrepresents the standard deviation of the model error. The function $\gamma(x,\theta)$can take various forms, for example,$\sum_{m=1}^{k} \theta_m h_m(x)$,where $h_m(x)$ is a set of suitable explanatory functions that may influence the capacity ofthe structural component (e.g., axial load ratio, shear span to depth ratio, spacing ofhoops).

The probabilistic capacity model accounts for both aleatory and epistemicuncertainties. Referring toEquation (1),aleatory uncertainties are present in the variables x, while epistemic uncertainties arepresent in the model coefficients θ**.** The error term $\sigma.\varepsilon$contains both aleatory andepistemic uncertainties.

BAYESIAN UPDATING APPROACH

The model coefficients (θ,σ) in Equation (1) are estimated by using the Bayesianupdating rule(Box et al., 1992):

$$f(\theta,\sigma) = \kappa\, L(\theta,\sigma) P(\theta,\sigma) \quad (2)$$

where $f(\theta,\sigma)$ denotes the posterior distribution representing our updated knowledgeabout (θ,σ); $L(\theta,\sigma)$denotes the likelihood function representing the objectiveinformation on (θ,σ) gained from a set of observations; $P(\theta,\sigma)$denotes the priordistribution reflecting our knowledge about(θ,σ) prior to obtaining the observations;and $\kappa = \left[\int L(\theta,\sigma)P(\theta,\sigma)d\theta\, d\sigma\right]^{-1}$is a normalizing factor.

The prior distribution may incorporate any information about(θ,σ) that is based on the previous experience or engineering judgment.$L(\theta,\sigma)$represents the likelihood of observing the experimental outcome for given values of the coefficients θ and σ. The formulation of the likelihood function depends on the type and form of the available data. Theoretically, the posterior distribution $f(\theta,\sigma)$ can be determined based on Equation (2) when the prior distribution and likelihood function have been formulated(Box et al., 1992).

The close form solution is valid for the case that the probabilistic model formulation isa linear function respect to θ, and all observations are classified as failure data.Assuming $\gamma(x,\theta) = \sum_{m=1}^{k} \theta_m h_m(x)$and *n* set experimental observations available, Equation (2) can be rewritten as:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} h_{11} & \dots & h_{k1} \\ \vdots & & \vdots \\ h_{1i} & \dots & h_{ki} \\ \vdots & & \vdots \\ h_{1n} & \dots & h_{kn} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_k \end{bmatrix} + \sigma. \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_i \\ \vdots \\ \varepsilon_n \end{bmatrix} \quad (3)$$

where$y_i = C_i - \hat{C}_i$represents the difference between the ith observed capacity and theestimated capacity of deterministic model at x_i; $h_{ki}$is the value of the kth explanatoryfunction at the ith observation.Equation (3) can also be expressed in the vector form(Gardoni, 2002):

$$y = H\theta + \sigma\varepsilon \quad (4)$$

Under the normality assumption on $\varepsilon$and a non-informative prior with θ, Box andTiao show that the marginal posterior distribution of $\theta$is a multivariate tdistribution.the mean vector of θ**,** M_θ, and the covariance matrix of θ **,** $\sum_{\theta\theta}$are(Gardoni, 2002):

$$M_\theta = (H'H)^{-1}H'y \quad (5)$$

$$\sum_{\theta\theta} = \frac{}{-2} s^2 (H'H)^{-1}$$

Where:

$$\hat{\theta} = (H'H)^{-1}H'y \quad (6)$$
$$\hat{y} = H\hat{\theta}$$
$$= n - k$$
$$s^2 = \frac{1}{-}(y - \hat{y})'(y - \hat{y})$$

Once the initial probabilistic model has been formed, a stepwise deletion procedure isused to reduce the number of terms in $\gamma(x,\theta)$to achieve a compromise between themodel simplicity (few correction terms) and the model accuracy (small σ). It is preferredto start the model assessment process with a comprehensive form of (x,θ) , and thensimplify the model by removing less important terms or combining terms that are closelycorrelated.

COLUMN CLASSIFICATIONAND DEFINITION OF FAILURE MODE

This study provides a probabilistic failure modeindex model which distinguishes failure modes for a given column.the onset of column failure is defined as 20% loss in the lateral strength,and three column failure modes (i.e., flexure failure, shear failure and flexure-shearfailure) are considered. The definitions of these three failure modes are schematicallyillustrated in Figure 1.

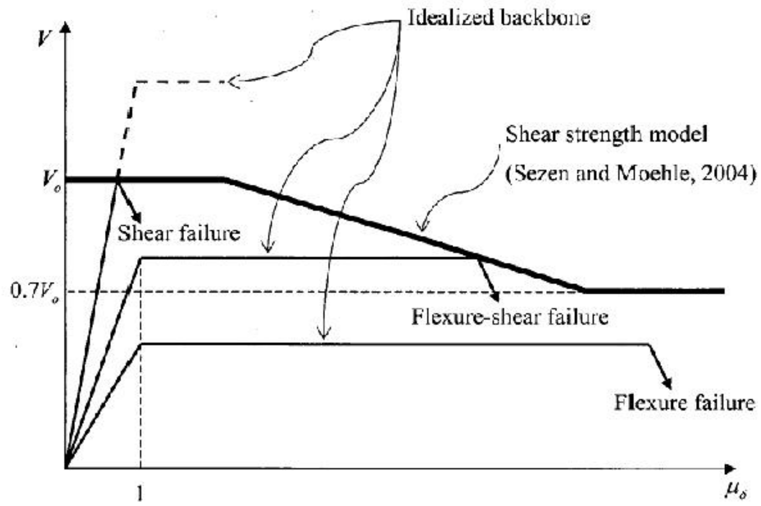

Figure1. Conceptual definition of column failure modes(Sezenet al.,2004)

Experimental Database

A database containing results of cyclic lateral-load tests ofreinforced concrete columns was compiled by researchers at the University ofWashington under the support of the Pacific Earthquake Engineering Research Center.The database consists of tests of 125 reinforced concrete columns.The dataaredivided into three categoriesin termsofspan to depth ratio.The key parameters of column failure are listed in table 1(Zhu, 1993).

Table1.Key parameters of column failure

1	2	3	4	5	6	7	8	9
ρ_l	$\frac{\rho_l f_{yl}}{f_c'}$	ρ''	$\frac{\rho'' f_{yt}}{f_c'}$	$\frac{\bar{s}}{d}$	$\frac{\bar{a}}{d}$	$\frac{\bar{v}}{\frac{\sqrt{f_c'}}{6} + \rho''\rho_{yt}}$	$\frac{\bar{p}}{A_g f_c'}$	$\frac{V_p}{V_0}$

where $a$is the shear span; d is the depth to the centerline of the outermost tensionreinforcement; s is the hoop spacing; $\rho_l = A_{sl}/bh$denotes the longitudinal reinforcementratio; A_{sl}denotes the total area of longitudinal reinforcement; b is the width of columnsection; h is the depth of column section; $\rho'' =$

A_{st}/bhdenotes the transversereinforcement ratio; A_{st}denotes the area of transverse reinforcement; $\nu = V_{eff}/bd$denotes the maximum nominal shear stress; p is the axial load; and $A_g = bh$ denotes thegross cross-sectional area of the column(Zhu, 1993).

Thefailure mode for each column in the selected database is based on the failure modesidentified by Camarillo.the relation between plastic shear demand and shear strengthprovides useful information in the determination of column failure modes. Here, thecolumn plastic shear demand is determined by its maximum moment capacity divided bythe shear span. The maximum moment capacity is computedthrough a moment-curvature analysis for the column's cross section using Manderconcrete constitutive model and Burns-Seiss steel constitutive model.The abbreviations of failure modes, 'F' , 'FS'and 'S' , denote flexure failure, flexure-shear failure and shear failure, respectively.Three integers, '1', '2' and '3', are assigned as failure mode indices (FM) to represent flexure failure, flexure-shear failure and shear failure, respectively(Zhu, 1993).

In order to investigate the dependence of the column failure modes on key parameters, nine parameters in Table 1 are interpreted as explanatory functions, $h_i(x)$, in the initial formulation of probabilistic failure mode index model. Note that some selected powers are applied to the column parameters to obtain better model prediction consistent with the experimental observation.The stepwise deletion procedure is used to reduce the number ofterms to achieve a compromise between the model simplicityand the model accuracy. the final model takes theform:

$$FM = \theta_1 + \theta_4(\rho'')^{\frac{-1}{4}} + \theta_7(\frac{a}{d})^{-2} + \theta_{10}\frac{V_p}{V_o} \quad (7)$$

Valuesof key parametersforthethree groups of dataare shownin Figures2 to4.

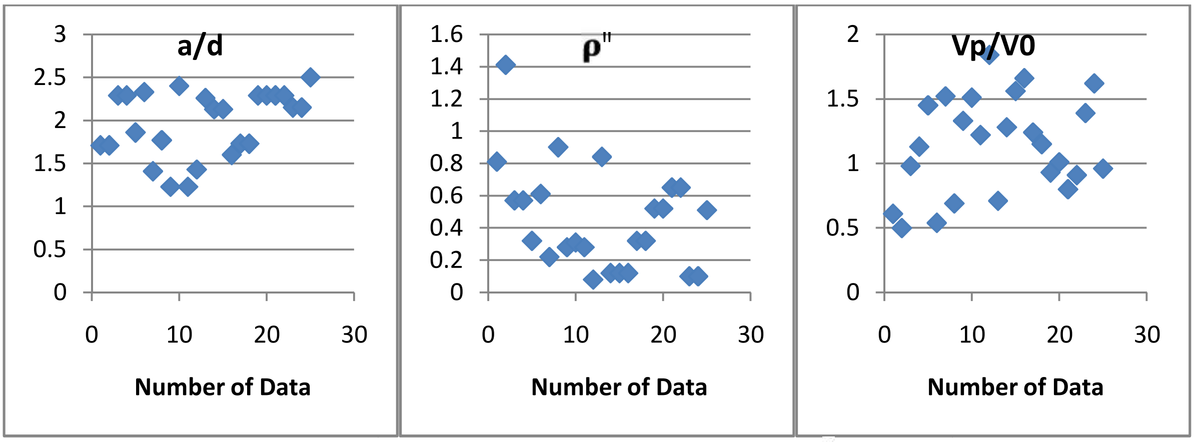

Figure 2. key Parameters of Column Failure ($1 < \frac{a}{d} \leq 2.5$)

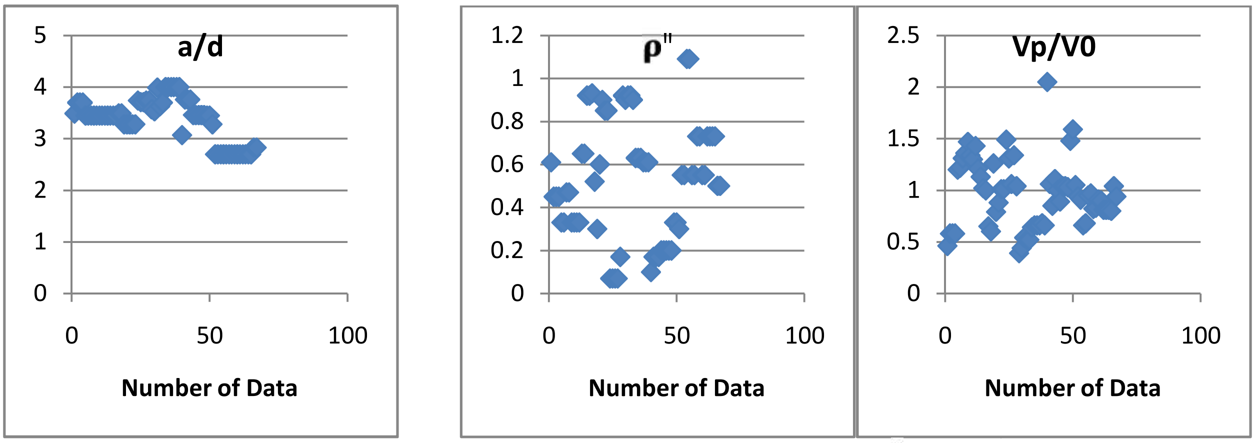

Figure 3. key Parameters of Column Failure ($2.5 < \frac{a}{d} \leq 4$)

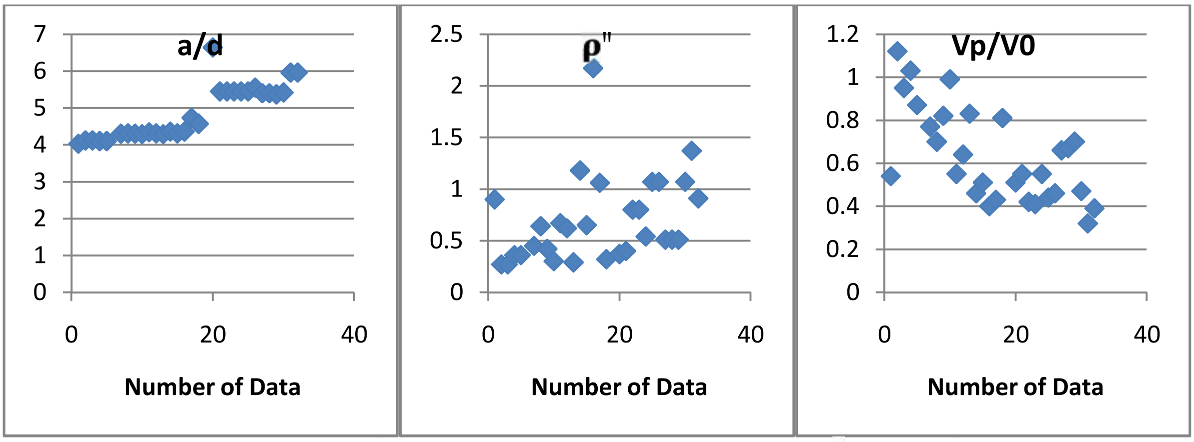

Figure 4. key Parameters of Column Failure ($4 < \frac{a}{d} < 7$)

Tables 2to4 list the posterior statistics of the model coefficients in Equation 7. Thisprobabilistic model identifies the most important parameters affecting the column failuremode, namely, transverse reinforcement ratio, aspect ratio, and shear demand-strengthratio. Note that the parameters, which have no clear relationship with column failuremodesare all eliminated through the modelassessment. Equation 7 is actually a probability density function of the failure modeindex; hence it can be used to assess the probability of each failure mode for a givencolumn through a reliability analysis.

Table 2. posterior statistics of the model coefficients ($1 < \frac{a}{d} \le 2.5$)

Coefficients	θ_1	θ_2	θ_3	θ_4
Mean	-0.027	0.223	2.667	-0.422
Standard Deviation	0.954	0.402	1.185	0.95

Correlation Coefficient	θ_1	θ_2	θ_3	θ_4
θ_1	1	-0.813	-0.273	0.504
θ_2	-0.813	1	0.092	-0.877
θ_3	-0.273	0.092	1	-0.227
θ_4	0.504	-0.877	-0.227	1

Table 3. posterior statistics of the model coefficients ($2.5 < \frac{a}{d} \le 4$)

Coefficients	θ_1	θ_2	θ_3	θ_4
Mean	0.159	0.132	-10.62	1.051
Standard Deviation	0.519	0.117	2.852	0.27

Correlation Coefficient	θ_1	θ_2	θ_3	θ_4
θ_1	1	-0.783	-0.738	0.185
θ_2	-0.783	1	0.393	-0.621
θ_3	-0.738	0.393	1	-0.246
θ_4	0.185	-0.621	-0.246	1

Table 4. posterior statistics of the model coefficients ($4 < \frac{a}{d} < 7$)

Coefficients	θ_1	θ_2	θ_3	θ_4
Mean	1.6	-0.345	-7.027	0.635
Standard Deviation	0.89	0.313	8.23	0.85

Correlation Coefficient	θ_1	θ_2	θ_3	θ_4
θ_1	1	-0.957	-0.649	0.823
θ_2	-0.957	1	0.517	-0.887
θ_3	-0.649	0.517	1	-0.707
θ_4	0.823	-0.887	-0.707	1

For the probabilistic failure mode index model in Equation (7), the mean predictionof FM for 3 groups of data are:

$$FM_{(mean)} = -0.027 + 0.223(\rho^{"})^{\frac{-1}{4}} + 2.667\left(\frac{a}{d}\right)^{-2} - 0.422\frac{V_p}{V_0}\left(1 < \frac{a}{d} \leq 2.5\right) \quad (8)$$

$$FM_{(mean)} = 0.159 + 0.132(\rho^{"})^{\frac{-1}{4}} - 10.62\left(\frac{a}{d}\right)^{-2} + 1.051\frac{V_p}{V_0}\left(2.5 < \frac{a}{d} \leq 4\right) \quad (9)$$

$$FM_{(mean)} = 1.6 - 0.345(\rho^{"})^{\frac{-1}{4}} - 7.027\left(\frac{a}{d}\right)^{-2} + 0.635\frac{V_p}{V_0}\left(4 < \frac{a}{d} < 7\right) \quad (10)$$

CONCLUSIONS

In this study, the methodology developed by Gardoni et al. (2002) is adopted to constructthe probabilistic capacity models for reinforced concrete columns. With a probabilistic capacity model, it is possible todetermine the probability of column failureby determining the area underthe probability density function. This methodology iscapable of incorporating a wide range of information, including existing deterministicmodels, laboratory test data, field observations, and engineering judgment.The database that consists of tests of spiral and rectangular reinforced concrete columns are divided into three categories in terms of span to depth ratio. The Bayesianupdating approach is used to assess the unknown model coefficients based on thecollected experimental database.

REFERENCES

Box GEP and Tiao GC (1992)Bayesian Inference in Statistical Analysis , Addison-Wesley, Reading, MA

Gardoni P (2002)Probabilistic Models and Fragility Estimates for Bridge Components and Systems, Ph.D.Dissertation, Department of Civil and Environmental Engineering, University of California, Berkeley

Gardoni P, Der Kiureghian A and Mosalam KM (2002)Probabilistic Capacity Models and Fragility Estimates for Reinforced Concrete Columns based on Experimental Observations, *Journal of Engineering Mechanics*, Vol. 128, No. 10, pp. 1024-1038

Sezen H and Moehle JP (2004) Shear Strength Model for Lightly Reinforced Concrete Columns, *Journal of Structural Engineering*, Vol. 130, No. 11, pp. 1692-1703

Zhu L(1993)Probabilistic Drift Capacity Models for Reinforced Concrete Columns, ph.D. thesis, Tongji University,china