LEVEL CROSSING APPROACH IN SPATIAL AND TEMPORAL DEPENDENCE IN EARTHQUAKE

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INTRODUCTION

In recent years, a vast majority of researches have been devoted to the study of seismic data that contain information about the complex events that lead to the earthquake [1]-[5].

We live in a world where random processes are ubiquitous. Although the random values of a stochastic process at different times may be independent random variables, but in most cases they are considered to indicate complicated statistical correlations. So, over the past decade, several different methods have been introduced to study the properties of the process. Spatial and temporal fluctuations of earthquake form time series. The purpose of the application of statistical mechanics is to describe the behavior of the time series that can help to better understanding of the stochastic processes. After that, we lie in an effort to reproduce or predict some experimental facts with extraction of useful information. An important question is what is the probability of obtaining or losing a certain level of return at different time intervals? For the first time, Jenson the *inverse method* in turbulence, and Simonsen et al. presented the *inverse statistics* to apply on similar financial data to answer the similar questions [6]-[7]. Inverse statistics suggests the inverting of the structure function equation, and instead of considering the average moments of distance between two points, gives the difference value between these two points. In the inverse statistics method is used of another popular technique called the *level crossing method (LC)*. In the level crossing method no scaling feature is explicitly required [9-14] and this is the main advantage of this technique for estimating the statistical information of the series. Level crossing based on stochastic processes that grasp the scale dependence of the time series.

What is the reason for the formation of level crossing? This method was developed for the study of a series of different insight. The memory, non-Gaussianity and waiting time (length) (an average time (length) interval that we should wait for an event to take place again [15]–[16]) could be measured by level crossing method. Since the fractional Gaussian noises are well-known examples, their comparison with empirical data can be used as a criteria to better understanding the results obtained from the level crossing method applied to unknown empirical data.

Here, the total amount that is designated as N^{+}_{tot}, which represents the total number of upcrossings of a series, reflects how memory plays role. To better assess the effects of memory, we have calculated shuffled counterparts of each underlying time series and compared their associated total number of so-called crossings, N^{+}_{sh}, with that given by their original time series N^{+}_{tot} to obtain the change percentage in the system. The autocorrelations are destroyed by the shuffling procedure.

LEVEL CROSSING ANALYSIS

For better understanding, we begin with a summary of the analysis of LC [9]–[14]. Consider the $\{x(t)\}$ series from time intervals between earthquake events as n_α^+ represents the number of positive difference crossings (upcrossings) at the level of $x(t)-\bar{x}=\alpha$ in time interval (Figure 1). For all time intervals, the mean value of n_α^+ is equal to $N_\alpha^+(T)$ [11]:

$$N_\alpha^+(T)=\langle n_\alpha^+(T)\rangle \qquad (1)$$

where $\langle , \rangle$ represents the ensemble average. For a homogeneous process (constant) the average upcrosings is in accordance with time interval T. As a result:

$$N_\alpha^+(T)=\nu_\alpha^+ T \qquad (2)$$

where ν_α^+ is the average frequency of crossing with positive slope at the same level of $y=x(t)-\bar{x}=\alpha$. Frequency parameter ν_α^+ could be deduced from the underlying probability density functions (PDF) of $y \equiv x(t)-\bar{x}$, and $y'=(y(t+\Delta t)-y(t))/\Delta t=\Delta y/\Delta t$, which is named as $P(y,y')$ [6, 10].

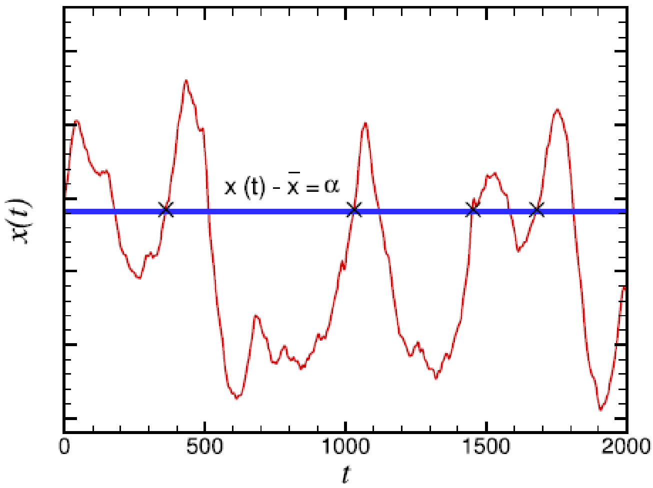

Figure 1. Schematic of upcrossing for an arbitrary level $x(t)-\bar{x}=\alpha$

Within the time interval Δt , the sample can only pass with positive slope at the level of $x(t)-\bar{x}=\alpha$ provided that it has the $x(t)-\bar{x}<\alpha$ property at the beginning of the interval. Furthermore there is a minimum difference at time *t*, if the level $x(t)-\bar{x}=\alpha$ is to be crossed in interval Δt depending on the value of $x(t)-\bar{x}$ at time *t*. Thus, it would be a positive crossing of $x(t)-\bar{x}=\alpha$ in the next interval Δt , if at time t:

$$x(t)-\bar{x}<\alpha \qquad \frac{\Delta[x(t)-\bar{x}]}{\Delta t}<\frac{\alpha-[x(t)-\bar{x}]}{\Delta t} \qquad (3)$$

As it was shown, ν_α^+ can be defined as the probability density function $P(y=\alpha\, , y')$ as follows:

$$\nu_\alpha^+=\int_0^\infty P(\alpha,y')y'\,dy' \qquad (4)$$

where $P(\alpha\, , y')$ is the joint probability density function $P(y\, , y')$ evaluated at $y=\alpha$.
Also, let us define the $N_{tot}^+(q)$ quantity as:

$$N_{tot}^+(q)=\int_{-\infty}^{+\infty}\nu_\alpha^+\left|\alpha-\bar{\alpha}\right|^q d\alpha. \qquad (5)$$

Where zero moment (with respect to ν_{α}^{+}) $q=0$ shows the total number of crossings with positive slope for return to earthquake magnitude. The moments $q<1$ will give information about the frequent events while moments $q>1$ are sensitive for the tail of events.

To investigate the effect of correlation and memory, N_{sh}^{+} is calculated which represents the total number of upcrossings in the time series when it is shuffled. Here, random permutation is used for shuffling the data. The autocorrelations are destroyed by the shuffling procedure. Hence, by comparing N_{tot}^{+} of the original data with that computed for the shuffled data set, N_{sh}^{+}, we can obtain the magnitude of correlations in the time series and this gives useful information about the time series. By comparing the difference between N_{tot}^{+} and N_{sh}^{+} (after shuffling), the memory of the time series can be determined. Smaller relative difference suggests that the time series is less correlated (anti-correlated). Under the influence of shuffling, the total number of crossings with positive slope N_{tot}^{+} increases (decreases) indicating the correlation in the underlying data (anti-correlation). In order to measure the value of memory (correlation and anti-correlation) in the data, the relative changes in the total number of upcrossings for original and shuffled data defined using N_{tot}^{+} as follows:

$$R \equiv \left| N_{sh}^{+} - N_{tot}^{+} \right| / N_{tot}^{+} \qquad (6)$$

APPLICATION ON EARTHQUAKE

We have studied Iran and California earthquakes from 1/1/1971 to 08/03/2013 [16]. For this purpose, the statistical correlations of earthquakes in these two regions have been compared. Earthquakes with magnitude greater than of 4 in Richter magnitude scale have been selected, and time series and the spatial series have been established by using time intervals and physical distances between the earthquakes, respectively.

Fig.2 shows the crossing with positive slope related to the obtained date (both original and shuffled data) for Iran and California earthquakes as can be estimated by equation (4).

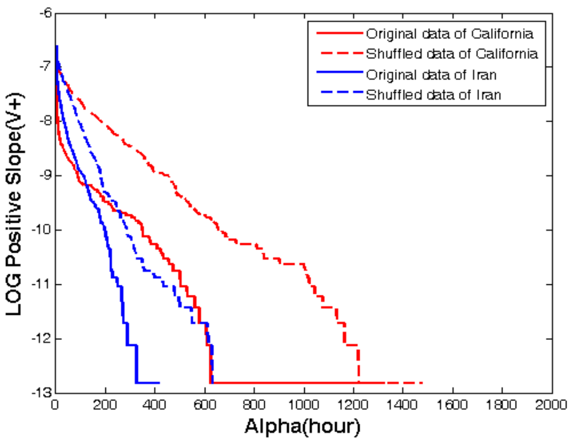

Figure 2. Plot of ν_{α}^{+} for original and shuffled date related to Iran and California earthquakes in time series.

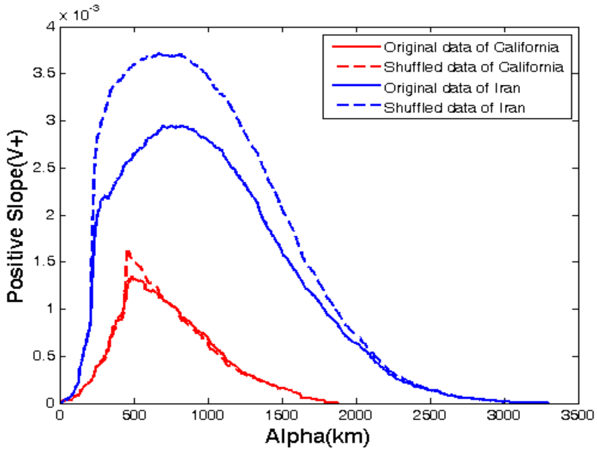

Figure 3. Plot of ν_{α}^{+} for original and shuffled date related to Iran and California earthquakes in spatial series.

The plots reveal the positive correlation of Iran and California earthquakes over time.

The level crossing for both original and shuffled data related to Iran and California earthquake have been depicted in Fig.3. From the figure, the earthquakes correlation is also evident in this case. However, the correlation is lower than that related to Iran earthquakes.

After examining the original and shuffled date for earthquakes occurred in Iran and California, now we will lie in an effort to compare earthquakes occurred these two regions. Earthquakes in these two regions with similar area have been investigated at the same time.

For the original data related to earthquakes occurred in Iran and California, the crossing with positive slope has been estimated using equation (4) in the time series and plotted in Fig.2. As it can be seen from the figure, the earthquake with smaller magnitude in Iran is more probable than California and the earthquake with large magnitude in California is more probable than Iran.

The estimated crossing with positive slope from equation (4) in the spatial series for the original data related to earthquakes occurred in Iran and California has been depicted in Fig.3. It is clear from the figure that the earthquakes occurred in Iran show a higher correlation. This means that the earthquakes affect and stimulate each other. In other word, the percentage of induced earthquakes in Iran is more than California.
The final number of crossings with positive slope have been calculated from equation (5) and plotted in Fig.4.

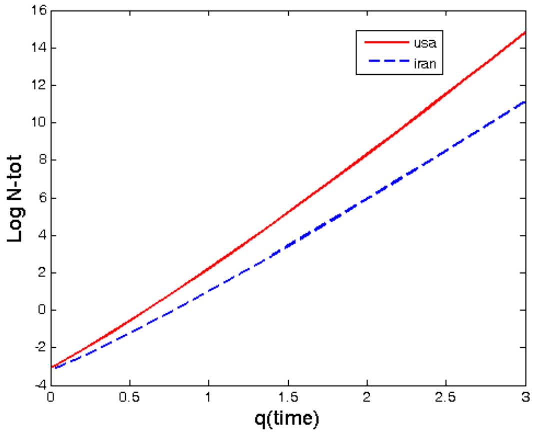

Figure 4. Plot of N^{+}_{tot} for Iran and California earthquakes in time series.

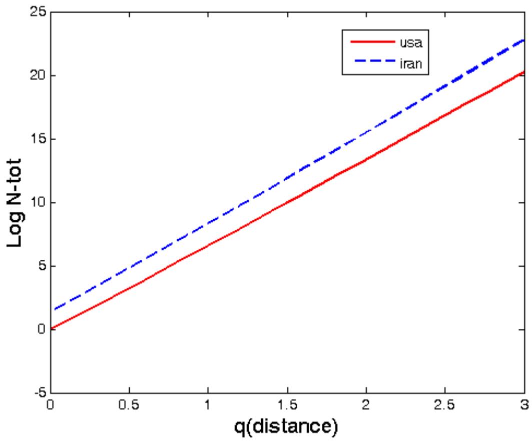

Figure 5. Plot of N^{+}_{tot} for Iran and California earthquakes in spatial series.

The slope of the plot for California is higher than that for Iran. This means that earthquake occurrence in California is more probable than Iran and in terms of time, the possibility increases with increase of earthquake magnitude. In addition, it is expected to occur a larger earthquake in California compared with Iran at far times. The final number of crossings with positive slope estimated from equation (5) in the spatial series have been calculated and plotted in Fig.5 for earthquakes in Iran and California locations. The slope of the plot is almost identical for Iran and California. This means that the earthquake occurrence in California and has a similar behavior in terms of spatial position.

The relative changes between Iran and California earthquakes for time series of q between 0 and 3 have been calculated Using equation (6) and plotted in Fig.6.

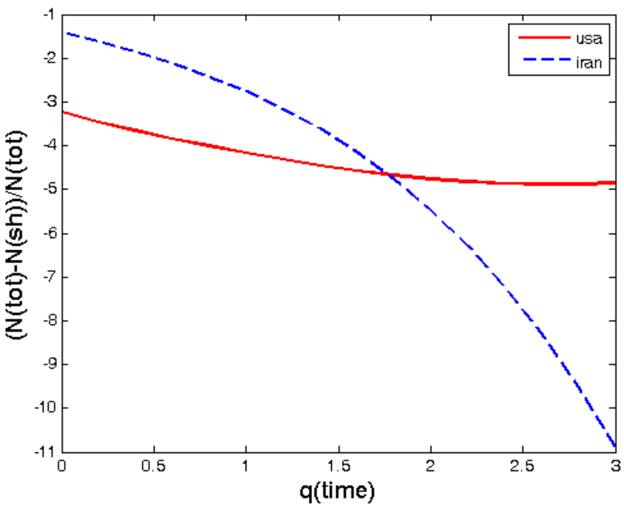

Figure 6. Plot of $\left|N^{+}_{sh} - N^{+}_{tot}\right| / N^{+}_{tot}$ for Iran and California earthquakes in time series.

CONCLUSION

In this paper the concept of level crossing analysis has been applied to Iran and California earthquakes. In level crossing method no scaling feature is explicitly required and this is the main advantage of this method in estimating the statistical information of the series. Level crossing based on stochastic processes that grasp the scale dependence of the time series. It is shown that the level crossing is able to detect the memory of series. This method with no required scaling feature is a powerful method in characterizing the time series. Considering all of the above discussions and results, we notice that Iran and California earthquakes are correlated over time and location. But in California the temporal correlation is lower than to Iran earthquakes.

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