DAMAGE DETECTION OF BRIDGES USING THE RESPONSE POWER SPECTRAL DENSITYFUNCTION AND SENSITIVITY EQUATION

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ABSTRACT

Structures undergo different types of loading during their lifetime. As these loads cause the performance of the structures to decay gradually, the urge of damage detection using nondestructive methods has been felt during the past two decades. In this study, a structural damage detection method is presented using measured power spectral density data. It uses the power spectral density function and the decomposed form of frequency response function to evaluate response sensitivity with respect to the change of stiffness parameters for finite element model updating. Damage is considered to be a reduction in structural stiffness parameters. For frequency domains introduced here, updated stiffness parameters are captured with high accuracy through solving the sensitivity equations by the least square approach. MATLAB software is used for the numerical analyses. The performance of this method is investigated through identifying the damage of a bridge truss structure considering different damage scenarios.

INTRODUCTION

Life and money losses due to an abrupt structural failure and also getting the best performance of a structure during its life time are important motivations for civil, mechanical and aerospace engineers to seek approaches for detecting the location of damaged parts of a structure as well as their damage intensity. Alongside non-destructive experimental methods such as acoustic or ultrasonic methods, magnetic field methods, eddy current methods or thermal field methods (Doebling et al., 1998), which require that the vicinity of the damage parts are known and accessible prior to the experimentation, some techniques are proposed which use data related to vibration characteristics of the damaged structure for damage detecting. The underlying idea of vibration-based damage identification methods is that the dynamic characteristics and response of structures are contingent on their mass, damping and stiffness properties, which are affected by damage. Therefore, one may extract the location and intensity of damage from vibration data of a damaged structure.

Vibration-based damage identification methods can be categorized by dynamic characteristics that they use and the approach that they employ to correlate different damage scenarios to dynamic properties. Some techniques investigate changes in natural frequencies (Fan and Qiao, 2011), mode shapes (Bell et al., 2007) ,mode curvatures(Wang et al., 2000), modal strain energy (Zhang et al., 1998) or dynamic flexibility (Gao and Spencer, 2006) to locate and quantify damage in a structure. There are other methods using frequency response functions towards that purpose (Sohn et al., 2004). Some approaches adopt a combination of the artificial neutral network (ANN) and principal modal components (PCA) to detect damage (Bamdara et al., 2014), whereas there are other methods that establish sensitivity equations as a relation between a damage scenario and the measured dynamic response (Araujo dos Santos et al., 2000). This study focuses on damage detecting methods based on model updating.

Larsson and Sas (Larsson and Sas, 1992) developed a model updating technique utilizing an exact dynamic condensation in which the objective function does not require the computation of the impedance matrix. They emphasized that the desired frequency range, which can be updated, is inherently limited by the condensation procedure. Incomplete measurements and their implication on the frequency response function (FRF) model updating formulation seem to restrict the method's ability to update larger finite element models (FEM). Araujo dos Santos et al. (Araujo dos Santos et al., 2000) proposed a damage detecting technique based on the sensitivity of orthogonality conditions structures to damage. They solved the sensitivity equations by the least square method. Modak et al. (Modak et al., 2002) employed a direct method and an iterative one for model updating using measured mode shapes and frequency response functions respectively. They reported that the iterative method results in predictions that are more accurate. Araujo dos Santos et al. (Araujo dos Santos et al., 2005) presented a damage identification technique based on FRF sensitivities. Their technique leads to a set of linear equations, which is solved using an algorithm that constrains the solution to be physically admissible. They performed a damage simulation and identification on a laminated rectangular plate.

Esfandiari et al. (Esfandiari et al., 2009) presented a FRF-based finite element model updating algorithm using the harmonic forced vibration FRF of the damaged structure. They correlated the changes in the FRF of structures due to damage to the changes of stiffness, mass and damping properties through damage sensitivity equations, which can be solved using the least square method. In another study (Esfandiari et al., 2010), Esfandiari et al. proposed a structural model updating technique using FRF data and measured natural frequencies of the damaged structure without any expansion of the measured data or reduction of the finite element model. They constructed sensitivity equations derived using the change of eigenvectors and measured natural frequencies of the damaged structure. They also expressed the change in eigenvector as the linear combination of the original eigenvectors.Zheng et al. (Zheng et al., 2015) presented an approach for structural damage identification based on the response power spectral density sensitivity analysis. They used stationary, random excitation with pseudo-excitation method (PEM) to obtain the dynamic response of structures and sensitivity of the power spectral density with respect to the damage parameters.

In the present study, a structural model updating approach is proposed using the power spectral density function and measured natural frequencies of the damaged structure without any expansion of the measured data or reduction of the finite element model. The sensitivity equations are constructed through expressing the change in power spectral density function in terms of the change in stiffness that is caused by the change in eigenvectors and measured natural frequencies of the damaged structure. The eigenvectors are also expressed as a linear combination of the original eigenvectors. The least square method is adopted to solve the sensitivity equation set through a proper weighting procedure.

THEORETICAL BACKGROUND

The power spectral density function of a system can be defined as:

$$S_{xx} = HS_{ff}H^T \tag{1)}$$

Where H is the frequency response function and S_{xx} and S_{ff} are output and input power spectral density functions, respectively (Newland, 1993).The frequency response function of a system with n degrees of freedom is defined as:

$$H(\omega) = [-\omega_r^2 M + i\omega_r C + K]^{-1} \tag{2)}$$

Where M, C and K are the mass, damping and stiffness matrices of the system, ω is the frequency of the excitation load and ω_r indicates the natural frequency of the rth mode. The response of the structure to a unit harmonic load can be expressed as below by spectral decomposition:

$$H_{il} = \sum_{r=1}^{n} \frac{\Phi_{ir}\ \Phi_{lr}}{\omega_r^2 - \omega^2 + 2i\omega\xi_r\omega_r} \tag{3)}$$

H_{il} represents the entire impedance matrix where i and l indicate the measurement and excitation points, respectively. Φ_r, ω_r and ξ_r represent mode shape, natural frequency and damping of the rth moderespectively.The power spectral density function of a damaged structure is defined as:

$$S_{xxd} = H_d S_{ff} H_d^{\,T} \tag{4}$$

that should be calculated. The rth mode shape of a structure after changes due to the damage is considered as:

$$\Phi_{rd} = \Phi_r + \delta\Phi_r \tag{5}$$

Where the index d represents the dependency of the parameter to the damaged structure and $\delta\Phi_r$ represents the change in rth mode shape due to the damage. Substituting Eq.5 in Eq.3 for the damaged structure leads to:

$$H_{ild} \cong \sum_{r=1}^{n} \frac{(\Phi_{ir} + \delta\Phi_{ird})(\Phi_{lr} + \delta\Phi_{lrd})}{\omega_{rd}^2 - \omega^2 + 2i\omega\xi_{rd}\omega_{rd}} \tag{6}$$

Assuming that the first nm natural frequencies of the damaged structure be available and neglecting the second order terms, Eq.6 can be rewritten as:

$$\begin{aligned} H_{ild} \cong & \sum_{r=1}^{nm} \frac{\Phi_{ir}\Phi_{lr}}{\omega_{rd}^2 - \omega^2 + 2i\omega\xi_{rd}\omega_{rd}} + \sum_{r=1}^{nm} \frac{\Phi_{ir}\delta\Phi_{lr}}{\omega_{rd}^2 - \omega^2 + 2i\omega\xi_{rd}\omega_{rd}} + \\ & \sum_{r=1}^{nm} \frac{\delta\Phi_{ir}\Phi_{lr}}{\omega_{rd}^2 - \omega^2 + 2i\omega\xi_{rd}\omega_{rd}} + \sum_{r=nm1}^{n} \frac{\Phi_{ir}\Phi_{lr}}{\omega_r^2 - \omega^2 + 2i\omega\xi_r\omega_r} \end{aligned} \tag{7}$$

As the measurement of the natural frequencies is feasible with high accuracy, the approximation of Eq.7 is realistic. The last term of the Eq.7 is related to unmeasured part of the natural frequencies that would compensate the effect of incomplete measurement. Moreover, the first term of this equation can be calculated using the properties and eigenvector of the intact structure and measured natural frequencies of the damaged structure. The other two terms of the equation which includes the mode shape changes, should be evaluated in an accurate way. To estimate these terms, the rate changes of modal vector of mode shapes of a structure is considered as a linear combination of eigenvectors for all modes. Thus, the mode shape changes in a structure caused by damage can be defined by the following first order series (Esfandiari et al., 2010).

$$\delta\Phi_{li} \cong \sum_{q=1}^{n} \alpha_{iq}\Phi_{lq} \tag{8}$$

Where,

$$\begin{cases} \alpha_{iq} = \dfrac{\Phi_q^{\,T}(\delta k - \omega_i^2\delta M)\Phi_i}{(\omega_i^{\,2} - \omega_q^{\,2})} & for\ \ q \neq i \\[2ex] \alpha_{ii} = -\dfrac{\Phi_i^{\,T}(\delta M)\Phi_i}{2} & for\ \ q = i \end{cases} \tag{9}$$

The damage is presumably related to the stiffness loss of the system only, and the actual damage caused by mass changes is negligible in reality. Therefore, α_{iq} is equal to:

$$\alpha_{iq} = \frac{\Phi_q^{\,T}(\delta k)\Phi_i}{(\omega_i^{\,2} - \omega_q^{\,2})} \tag{8}$$

As the expansion in Eq.10 does not require the derivation of the denominator in Eq.3, it is barely nonlinear in comparison to using Taylor series expansion.Substituting Eq.10 in Eq.7, leads to:

$$\begin{aligned} H_{ild} = & \sum_{r=1}^{nm} \frac{\Phi_{ir}\Phi_{lr}}{\omega_{rd}^2 - \omega^2 + 2i\omega\xi_{rd}\omega_{rd}} + \sum_{r=nm+1}^{n} \frac{\Phi_{ir}\Phi_{lr}}{\omega_d^2 - \omega^2 + 2i\omega\xi_d\omega_d} + \sum_{r=1}^{nm}\sum_{q=1}^{n} \frac{\Phi_{ir}(\Phi_q^{\,T}\delta k\Phi_i)\Phi_{lq}}{(\omega_{rd}^2 - \omega^2 + 2i\omega\xi_{rd}\omega_{rd})(} \\ & + \sum_{r=1}^{nm}\sum_{q=1}^{n} \frac{(\Phi_q^{\,T}\delta k\Phi_i)\Phi_{iq}\Phi_{lr}}{(\omega_{rd}^2 - \omega^2 + 2i\omega\xi_{rd}\omega_{rd})(\omega_i^{\,2} - \omega_q^{\,2})} \end{aligned} \tag{11}$$

The first two terms of the right hand side of Eq.11 is known using natural frequencies of damaged structure and the eigenvectors of the intact structure and $H_{iid}$is measured. The known part can be expressed as:

$$\bar{H}_{il} = \sum_{r=1}^{nm} \frac{\Phi_{ir}\Phi_{lr}}{\omega_{rd}^2 - \omega^2 + 2i\omega\xi_{rd}\omega_{rd}} + \sum_{r=nm+1}^{n} \frac{\Phi_{ir}\Phi_{lr}}{\omega_d^2 - \omega^2 + 2i\omega\xi_d\omega_d} \qquad 12)$$

The unknown parts of Eq.7 can be expressed as:

$$\Delta H_{il} = \sum_{r=1}^{nm}\sum_{q=1}^{n} \frac{\Phi_{ir}(\Phi_q{}^T\delta k\Phi_i)\Phi_{lq}}{(\omega_{rd}^2 - \omega^2 + 2i\omega\xi_{rd}\omega_{rd})(\omega_i{}^2 - \omega_q{}^2)} + \sum_{r=1}^{nm}\sum_{q=1}^{n} \frac{(\Phi_q{}^T\delta k\Phi_i)\Phi_{iq}\Phi_{lr}}{(\omega_{rd}^2 - \omega^2 + 2i\omega\xi_{rd}\omega_{rd})(\omega_i{}^2 - \omega_q{}^2)} \qquad 13)$$

Equation7 can be rewritten as follows:

$$H_{ild} = \bar{H}_{il} + \Delta H_{il} \qquad 14)$$

Therefore, Eq.4 can be defined as:

$$S_{xxd} = (\bar{H}_d + \Delta H_d)S_{ff}(\bar{H}_d + \Delta H_d)^T \qquad 15)$$

By separating the known terms of Eq.15 and neglecting the second order terms one obtaines:

$$\Delta S_{xx} = \Delta H_d S_{ff}\bar{H}_d +^T \bar{H}_d S_{ff}\Delta H_d{}^T \qquad 16)$$

Where

$$\Delta S_{xx} = S_{xxd} - \bar{H}_d S_{ff}\bar{H}_d{}^T \qquad 17)$$

The stiffness matrix of each element of a structure can be defined as:

$$K_e = A_e.P_e.A_e{}^T \qquad 18)$$

Where $A_e$is the eigenvector of nonzero eigenvalues of the stiffness matrix and P_e is corresponding nonzero eigenvalue of the stiffness matrix. The matrix A_e for the whole structure can be defined in global coordinates as the following:

$$A = \sum_{i=1}^{ne} T^T{}_{ei}.A_{ei} \qquad 19)$$

Where T_{ei} is the transformation matrix of the i^{th}element from local to global coordinates and ne is the total number of elements. Thus, the stiffness matrix of the structure in global coordinates is defined as:

$$K = \sum_{i=1}^{ne} T^T{}_{ei}.A_{ei}.P_{ei}.A^T{}_{ei}.T_{ei} = A.P.A \qquad 20)$$

The stiffness matrix of the damaged structure can be defined as:

$$K_d = K + \delta K = A.(P + \delta P).A^T \qquad 21)$$

Where δP is the change of elemental stiffness due to the damage. The δK can be expressed as:

$$\delta K = A.\delta P.A^T$$
22)

considering Eq.13 and Eq.22, ΔS_{xxd} can be defined as:

$$\Delta S_{xxil}(\omega) = \Bigg(\left[\sum_{r=1}^{nm}\sum_{q=1}^{n}\frac{\Phi_{ir}(\Phi_q{}^T.A.diag(A^T.\Phi_r))\Phi_{lq}}{(\omega_{rd}^2-\omega^2+2i\omega\xi_{rd}\omega_{rd})(\omega_i{}^2-\omega_q{}^2)} + \sum_{r=1}^{nm}\sum_{q=1}^{n}\frac{\Phi_{iq}(\Phi_q{}^T.A.diag(A^T.\Phi_r))\Phi_{lr}}{(\omega_{rd}^2-\omega^2+2i\omega\xi_{rd}\omega_{rd})(\omega_i{}^2-\omega_q{}^2)}\right].S_{ff}.\left[\sum_{r=1}^{nm}\frac{\Phi_{ir}\Phi_{l}}{\omega_{rd}^2-\omega^2+2i\omega} \sum_{r=nm+1}^{n}\frac{\Phi_{ir}\Phi_{lr}}{\omega_d^2-\omega^2+2i\omega\xi_d\omega_d}\right]^T + \left[\sum_{r=1}^{nm}\frac{\Phi_{ir}\Phi_{lr}}{\omega_{rd}^2-\omega^2+2i\omega\xi_{rd}\omega_{rd}} + \sum_{r=nm+1}^{n}\frac{\Phi_{ir}\Phi_{lr}}{\omega_d^2-\omega^2+2i\omega\xi_d\omega_d}\right].S_{ff}.\left[\sum_{r=1}^{nm}\sum_{q=1}^{n}\frac{\Phi_{ir}(\Phi_q{}^T.A.diag(A^T.\Phi_r))\Phi_{lq}}{(\omega_{rd}^2-\omega^2+2i\omega\xi_{rd}\omega_{rd})(\omega_i{}^2-\omega_q{}^2)} + \sum_{r=1}^{nm}\sum_{q=1}^{n}\frac{\Phi_{iq}(\Phi_q{}^T.A.diag(A^T.\Phi_r)}{(\omega_{rd}^2-\omega^2+2i\omega\xi_{rd}\omega_{rd})(\omega_i}\Bigg).\delta P$$
23)

Where the operator "$diag$" indicates the entries of a diagonal matrix as a vector and vice versa. The coefficient of δP is the sensitivity matrix of the i^{th} degree of freedom when subjected to the applied unit load at the l^{th}degree of freedom. Eq. 23 can be rewritten as:

$$\Delta S_{xx}(\omega) = S(\omega).\delta P$$
24)

where $S(\omega)$ is the total sensitivity matrix, and $\delta P$is the vector of all stiffness changes. It is possible to solve Eq.24 with different methods like least square method (LS), non-negative least square (NNLS)and singular value decomposition method (SVD). In this study the LS method is used to solve the equation. Since it is possible that the LS method be dominated by equations with larger coefficients, a weighting technique is needed to prevent the information of some equations to be overshadowed by some others. Therefore, to improve the quality of the predicted damage, a weighting technique can be applied to the equations. There are different methods for weighting the equations. Kwon and Lin (30) state that weighing the sensitivity equation by ω^{-1} decreases inaccuracy of the finite element modeling at higher frequencies. The sensitivity of the PSD increases in higher frequency ranges. However, due to larger approximations at the higher frequencies, the weight of the sensitivity equations in this range must be decreased. Thus, in this study each sensitivity equation is multiplied to the associated $\omega^{-0.5}$ to smooth the effect of higher frequencies.

Another important issue in sensitivity-based model updating methods is the noise polluted the experimental data which may cause convergence to a local minimum. At proximity of natural frequencies of the damaged structure this noise can have serious effects on the response because of the term $\omega_d^2 - \omega^2$ in equations. There are other types of errors which can cause problems in such methods like mass modeling errors which means the error associated with the mass of modeled intact structures and the errors of measuring natural frequencies. Therefore, the proposed methods in this area should be robust to these errors.

NUMERICAL RESULTS

The finite element model of a 2-D truss consisting of 35 elements and 16 DOFs is considered. The elements are made of steel with Young's modulus of 20 Mpa and cross sectional areas given in Table 1. Also, the kinematic DOFs of the truss model are shown in Fig.2.

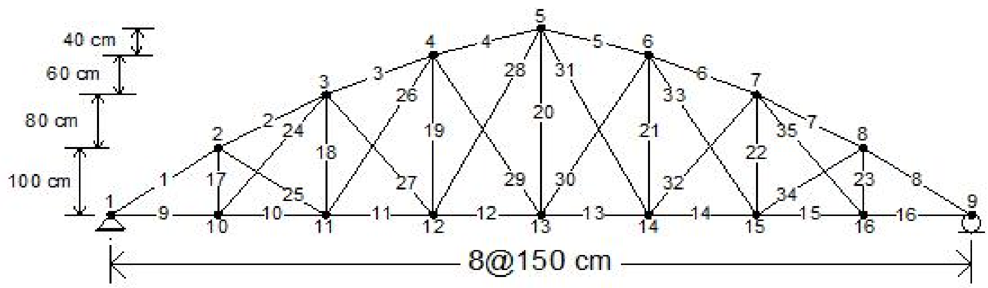

Figure 1. Geometry of truss model

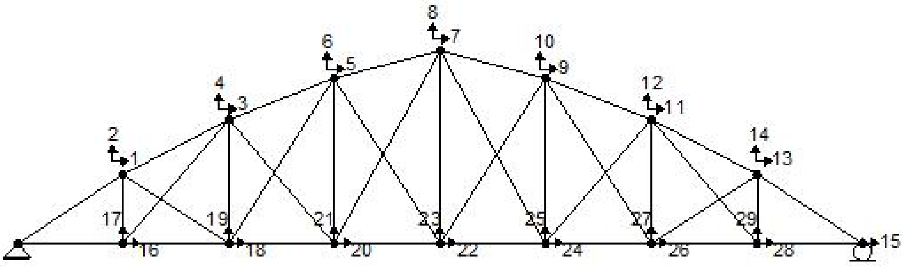

Figure 2. Free Degrees of freedom of the truss

Table 1. Cross sectional area of the truss elements

Element Number	Area (cm^2)
1-8	1.8
9-16	1.5
17-23	1
24-35	1.2

The location, severity and the number of damage elements can affect the results of a damage detecting process. Therefore, several damage scenarios listed in Table2, are considered to investigate the strength of the procedure presented in this study. In practical cases, the power spectral density function of the damaged structure is acquired by an experimental setup. Here, the finite element method is adopted to simulate the power spectral density function while including some virtual measurement errors in regard to probable uncertainties. In each scenario, a single harmonic excitation is applied to DOFs 9, 13, 15, 17 and 19 (the corresponding entity on the diagonal of the matrix Sff is set as 1 while the rest remain zero) for each load case. Also, DOFs 7, 9, 11, 14, 17, 18, 27 and 28 are chosen to be the locations where the response is being monitored.

Table 2. Damage scenarios

Scenario No.		Element	number	and damage	Percentage	
1	Element number	5	14			
	Damage percentage	30	50			
2	Element number	4	16	23		
	Damage percentage	40	50	60		
3	Element number	3	11	32	35	
	Damage percentage	30	30	30	30	
4	Element number	7	19	27		
	Damage percentage	30	40	30		
5	Element number	2	9	13	29	33
	Damage percentage	30	40	50	40	30
6	Element number	6	20			
	Damage percentage	40	70			

The frequency ranges of the excitation that is selected to construct the sensitivity equations are summarized in Table 3. These frequency ranges are the ones in which the left hand side of the sensitivity equation ΔS_{xx} (Eq. 17) is large enough so that the chance of successful predictions of the damage location and its severity improves.

Table 3.Selected frequency ranges for model updating

Damage Scenario	1	2	3	4	5	6
Frequency Range	218-224	218-223	221-226	222-227	209-214	250-255
	228-234	227-232	230-235	231-236	218-223	259-264
	288-294	301-306	294-299	292-297	293-298	303-308
	298-305	310-315	303-308	302-304	302-307	312-317
	313-319	319-324	314-319	308-313	313-318	332-337
	323-330	328-333	323-328	329-334	322-327	341-346

As mentioned before, natural frequencies and the power spectral density function, which are obtained by experimentation in reality, are simulated numerically in this study. In practical cases, there are some errors in measurement and data processing that can have adverse effects on the results. Here, a uniform random 5% error is included in the power spectral density function computed by the finite element method fortaking into account the existing inaccuracies of experimental data. The results of damage detection using the proposed formulation are depicted in Figs. 3-8.

Figs. 3-8 indicate that the performance of the proposed approach for detecting the location and the severity of damage in the structure is promising. In order to gain a better evaluation of the accuracy of results, some indices quantifying the discrepancies are defined. An average value of the absolutediscrepancies between the true damage values ($\delta\bar{P}_t$) and the predicted damage ones ($\delta\bar{P}_p$) is obtained by the mean sizing error (MSE)(Kim and Stubbs, 1995):

$$MSE = \frac{1}{ne}\sum_{e=1}^{ne} |\delta P_{ae} - \delta P_{pe}| \tag{25}$$

In addition, the relative error and the closeness index defined as,

$$RE = \frac{\sum_{e=1}^{ne}|\delta P_{ae}| - \sum_{e=1}^{ne}|\delta P_{pe}|}{\sum_{e=1}^{ne}|\delta P_{ae}|} \quad , \qquad CI = 1 - \frac{|\delta P_p - \delta P_a|}{|\delta P_a|} \tag{26}$$

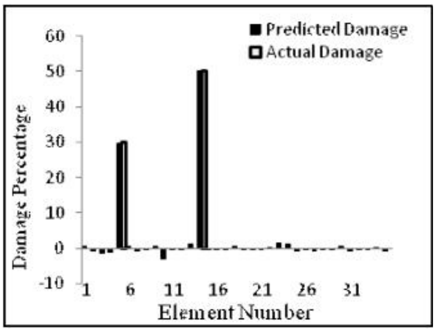

Figure 3. Actual and predicted damage of scenario 1 using noisy data

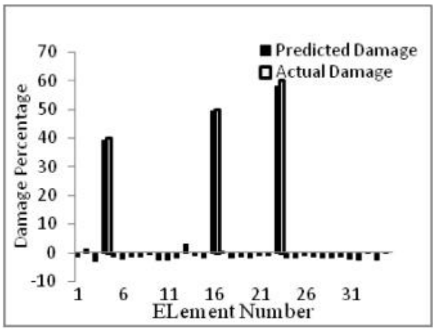

Figure 4. Actual and predicted damage of scenario 2 using noisy data

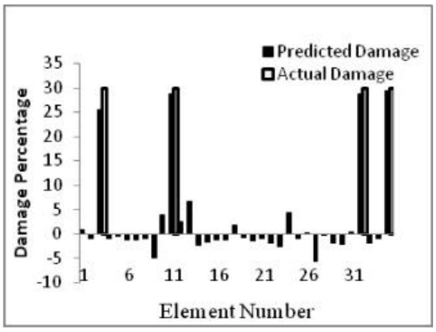

Figure 5. Actual and predicted damage of scenario 3 using noisy data

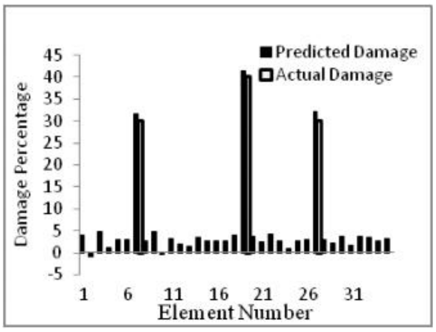

Figure 6. Actual and predicted damage of scenario 4 using noisy data

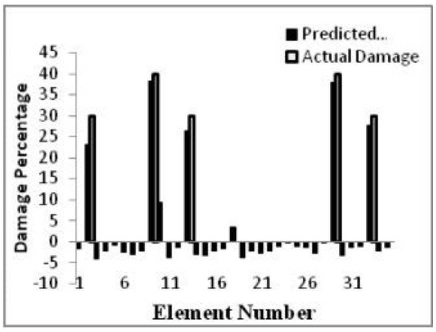

Figure 7. Actual and predicted damage of scenario 5 using noisy data

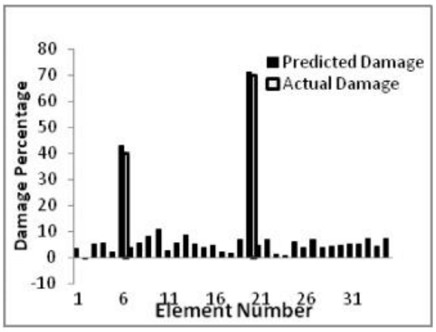

Figure 8. Actual and predicted damage of scenario 6 using noisy data

represent the distance between the true and estimated damage parameter vectors. An element is identified as damaged if $|\delta\bar{P}_p| > 2 \times MSE$(Kim and Stubbs, 1995). Clearly, the smaller values of the MSE and RE and larger values of the CI show better results. The damage indices of all damage scenarios are presented in Table 4.

Table 4.Comparison of damage indices

Damage Scenarios	MSE	RE	CI
1	0.006	0.0025	0.9917
2	0.015	0.0122	0.9828
3	0.0189	0.0071	0.9747
4	0.0275	0.0276	0.9697
5	0.025	0.0187	0.9685
6	0.0472	0.458	0.9462

In order to investigate robustness of the method, standard deviations of the predictedunknown parameters are also evaluated. Small standard deviations indicate that the results are less scattered and more reliable. The coefficient of variations (COV) of the estimated unknown parameters corresponding to cases shown in Figs. 3-8 are plotted in Figs. 9-14 respectively.In some cases a uniform random 0.5% error is introduced in the natural frequencies of the damaged structure which are computed numerically to investigate the effect of probable inaccuracies occurring in experimental values. The results are summarized in Table 5.

Table 5. Comparison of damage indices considering uncertainties of measured natural frequencies

Damage Scenarios	MSE	RE	CI
3	0.0467	0.0479	0.9458
6	0.0163	0.0066	0.9771

In addition to errors regarding experimental data, there may also be some discrepancies between the assumptions used in the mathematical model of the structure. A uniform random 5% error is applied to the specific weight of elements in computing their mass matrices in order to involve the uncertainties existing in mathematical modeling of the structure. Results of the damage detecting process of two cases (3 and 7) by inaccurate mass matrices are presented in Table 6.

Table 6. Comparison of damage indices considering uncertainties of the mass matrix

Damage Scenarios	MSE	RE	CI
3	0.0573	0.0035	0.9166

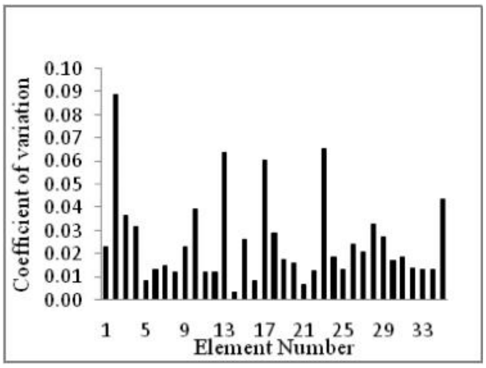

Figure 9. Coefficient of variations of estimated parameters of scenario 1

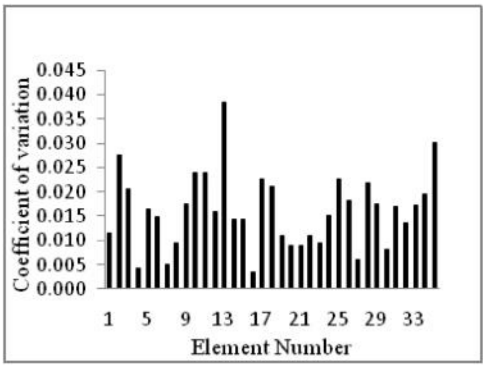

Figure 10. Coefficient of variations of estimated parameters of scenario 2

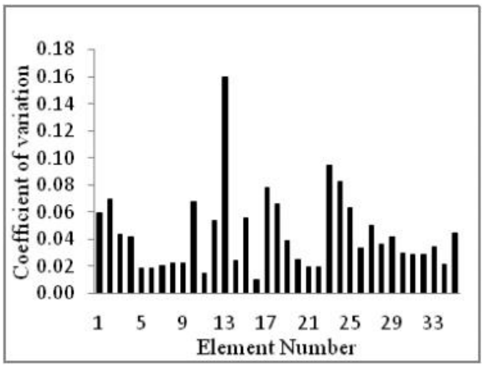

Figure 11. Coefficient of variations of estimated parameters of scenario 1

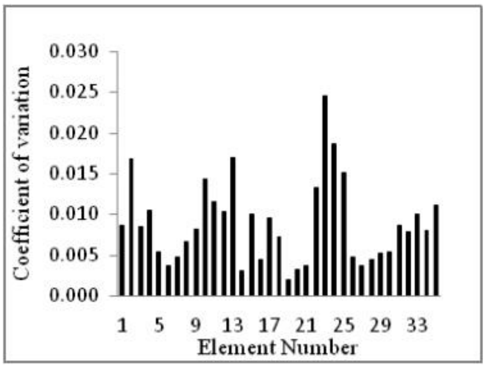

Figure 12. Coefficient of variations of estimated parameters of scenario 2

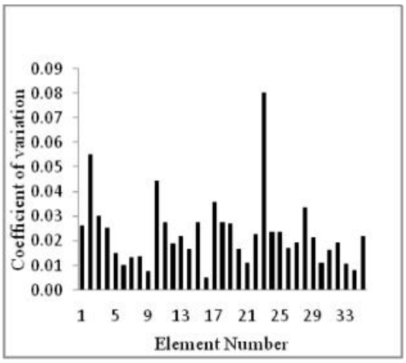

Figure13.Coefficient of variations of estimated parameters of scenario 5

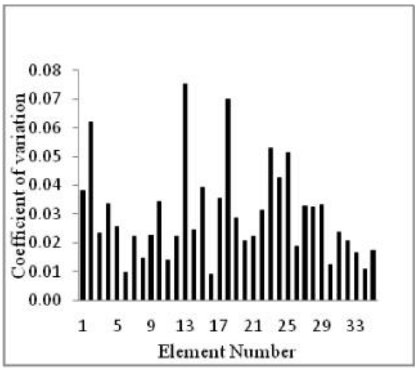

Figure14.Coefficient of variations of estimated parameters of scenario 6

CONCLUSION

A structural damage detection method is presented using the power spectral density function and measured natural frequencies. The damage is considered as the change of stiffness parameters corresponding to elements. The change of the power spectral density function is expressed in terms of the change in mode shapes and natural frequencies. The sensitivity equations are established through correlating the change of the power spectral density function of the structure to damage in elements. Sensitivity equations are solved by the Least Square method to compute change of structural parameters. Results of a truss modelshow the ability of this method to identify location and severity of parameters change at the elemental level in a structure.

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