STRUCTURAL DAMAGE DETECTION USING BASIS PURSUIT AND EARTHQUAKE TIME HISTORY ANALYSIS

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ABSTRACT

In this paper, a new method is proposed for structural damage detection using Basis Pursuit and earthquake time history analysis. The sensitivity matrix of structural responses with respect to elemental damage is established and solved by a new spars recovery method named Basis Pursuit (BP). The Simplex method is chosen as a linear programming method to solve the system of equations. The process detects the damage locations and extents using the time-history responses of structure. The efficiency of the proposed method is investigated using Monte Carlo simulation. The results are compared with Pseudo-inverse method results. The proposed method is applied to a cantilever beam and a planar truss. The proposed method efficiently detects the damage locations and extents. The running time of the proposed method is compared with some other methods. The simulation results demonstrate the roubostness and efficiency of the proposed method.

INTRODUCTION

In recent years a great deal of work has been carried out on development of methods to detect the location and extent of structural damage. A class of non-destructive damage detection methods is the vibration based method using dynamic responses such as natural frequencies and mode shapes (Doebling et. al., 1996; Sohn et. al., 2004). Gue and li (2009) proposed a two-stage method to determine the location and extent of multiple structural damages. At the first stage, the damaged sites were localized using the evidence theory for frequencies and mode shapes data. At the second one, micro search genetic algorithm of elitists was employed to improve search efficiently.

Some common methods of solving the system of equations are pseudo-inverse method, least square method and non-negative least square method. Recently, some new methods are used to solve the problem such as Orthogonal Matching Pursuit (OMP) (Pati et. al., 1993) and Basis Pursuit (BP) (Chen et. al., 2001). The sparse recovery characteristics of BP are considered by Chen et. al. (2001) and Donoho and Elad (2006). The matching pursuit (liu et. al., 2002) and basis pursuit (Yang et. al., 2003) were applied to identify damage bearings effectively. Yang et al. (2007) presented a procedure to detect the fault of rolling element bearings which combined the basis pursuit and a Feed Forward Neural Network (FFNN) classifier. Also, the results of BP and MP methods for fault diagnosis of rolling element are compared using vibration analysis. The comparison demonstrated that Basis Pursuit feature-based fault diagnosis is more accurate than Matching Pursuit feature-based fault diagnosis in detecting the faults. Ryan et. al. (2009) proposed a different approach to detect the damage using FRFs and OMP method. They calculated the residual damage based on FRF data from a possibly damaged system and a Finite Element Model (FEM) of the healthy system. This residual damage builds a system of equations to relate the damage residual to the actual damage on each element. For the systems subject to the noise, OMP method is used to overcome the issues. A two stage method using basis pursuit and genetic algorithm was proposed (Gerist et. al., 2012). In their method, the sensitivity matrix of structural responses with respect to elemental damage is established to be solved by Basis Pursuit (BP) using the frequencies of structure. In the final stage, they used the continuous genetic algorithm optimization method to improve the solution.

In this paper, a new method is presented to detect the damage extents and locations. First, the sensitivity matrix of structural responses with respect to elemental damaging is established using finite difference method with random intervals to be solved by BP method. The simplex method is chosen as a linear programming method to solve the system of equations. The efficiency of the proposed method is investigated using Monte Carlo simulation. The proposed method is applied to a cantilever beam and a planar truss.

DAMAGE DETECTION

The structural damage detection problems can form a set of equations. To solve the equations, a set of damage variables should be found to equalize the analytical and measured responses of the structure in an optimal way. The equation is considered as follows:

$$\mathbf{R}_d = \mathbf{R}(\mathbf{X}) \Rightarrow \mathbf{X} = ? \tag{1}$$

Where, $\mathbf{X} = (x_1, x_2, ..., x_n)^{\mathrm{T}}$ is called the design variables and n is the number of structural elements. x_i is the ratio of damaged element stiffness to the intact one of the ith element that is named damage ratio. $\mathbf{R}_d = (r_{\mathrm{d}1}, r_{\mathrm{d}2}, ..., r_{dm})^{\mathrm{T}}$ is the vector of m structural responses of measured damage structure. Vector of m responses of analytical model is shown by $\mathbf{R(X)} = (r_1(\mathbf{X}), r_2(\mathbf{X}), ..., r_m(\mathbf{X}))^{\mathrm{T}}$. Eq. (1) is approximated by the first order method as follows:

$$\mathbf{R}_d = \mathbf{R}(\mathbf{X}) = \mathbf{R}_h + \frac{\partial \mathbf{R}}{\partial \mathbf{X}} \Delta \mathbf{X} + ... \Rightarrow \mathbf{R}_d - \mathbf{R}_h = \Delta \mathbf{R} \cong \mathbf{S} \Delta \mathbf{X} \tag{2}$$

in which, $\mathbf{R}_h$, $\Delta \mathbf{X}$ and $\mathbf{S}$ are response vector of the healthy structure, damage vector change and sensitivity matrix of structural responses, respectively. This information is referred to as sparse approximation (Chen et. al. 2001). An explosion of interest in regularization via sparsity constraints has been happened in the last two decades. Hence, some approximate solutions are considered for linear systems in which the unknown has few nonzero entries relative to its dimension. So, the main object of this paper is to find the sparsest solution of the linear system.

BASIS PURSUIT

In classical theory of linear algebra, when measurements of an equation are fewer than unknowns, the problem is undetermined and the solution is generally not unique. The mathematical expression of the set of linear equations is as follows:

$$\mathbf{A} \times \mathbf{x} = \mathbf{b} \tag{3}$$

where $\mathbf{b}$ is a vector of interest $\mathbf{b} \in \mathbf{R}^m$ and $\mathbf{x}$ is a subspace of $\mathbf{R}^n$. $\mathbf{A}$ is a matrix of $\mathbf{R}^{m \times n}$ and is named dictionary matrix when each columns of it is normalized. So the aim is to identify a solution with minimal support using sparse representation. To solve Eq. (1), a model can be found when $\mathbf{x}$ is known to be S-sparse for some $1 \le S \le n$, which means that at most S coefficients of $\mathbf{x}$ can be non-zero. In principle, only S measurements are required to reconstruct $\mathbf{x}$ rather than n.

Three criteria are related to the notion of sparsity: the l_0, l_1 and the l_2 norms of **x.** The l_0 norm is the unique sparsest solution and it is evaluated as bellow:

$$x = \operatorname{argmin}_{x:\mathbf{AX}=\mathbf{b}} \|\mathbf{x}\|_{l_0} \quad (4)$$

where, $\|\mathbf{x}\|_{l_0}$ is the number of non-zero values of **x** and can be considered as follows:

$$\|\mathbf{x}\|_{l_0} = \sum_{i=1}^{n} |x_i|^0 = \{1 \le i \le n : x_i \ne 0\} \quad (5)$$

Unfortunately, the l_0 sparse approximation is at least as hard as a general constraint satisfaction problem even with no restrictions on **A** and **b**. Hence, Eq. (1) requires non-polynomial time to be solved and it is computationally intractable. In fact, the problem is NP-hard in general because l_0 minimization is not a convex optimization problem and its complexity exponentially increases with the number of dictionary matrix columns (Natarajan, 1995; Davis et. al., 1997). The l_2-norm which is in fact the least square solution is evaluated by:

$$x' = \operatorname{argmin}_{x:\mathbf{AX}=\mathbf{b}} \|\mathbf{x}\|_{l_2} = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2} \quad (6)$$

where, x_i is the ith component of vector **x**. Computation of l_2 minimization is efficiently less time-consuming than l_0. The result of l_2 minimization, x', doesn't mach the one of l_0, *x*. Recent studies of this problem show that it can be solved for some cases by either greedy or convex programming approaches. A new convex and tractable approach is presented as Basis Pursuit which uses l_1 minimization to solve the problem (Chen et. al. 2001). The l_1-norm is the summation of the absolute value of **x** components that can be expressed mathematically as:

$$x^* = \operatorname{argmin}_{x:\mathbf{AX}=\mathbf{b}} \|\mathbf{x}\|_{l_1} = \sum_{i=1}^{n} |x_i| \quad (7)$$

The l_1 minimization can be solved in polynomial time. The sparse vector **x** is reconstructed exactly from Eq. (1) and Eq. (7). So the l_1 norm exactly recover the l_0 one and x^* is equal to *x*. BP is closely connected with Linear Program (LP) and it works whenever the dictionary matrix is sufficiently incoherent. It roughly means that dictionary matrix entries are uniform in magnitude. The Basis Pursuit (BP) method is used for linear equations while the structural damage detection problem is a set of nonlinear equations. Considering an admissible approximation, the damage detection problem can be solved as a linear problem by BP.

The sensitivity matrix of the structure is calculated by the Finite Difference Method.

CASE STUDY

In this part, the proposed method is verified by two different case studies. The damage is simulated by reduction in Young modulus of the elements. The Bam earthquake and average acceleration method is used for time-history analysis. To demonstrate the efficiency and accuracy of the proposed method, the proposed method is investigated by Monte Carlo simulation and the results are compared with Pseudo-inverse method (PI). Also, run time of the proposed method is compared with CGA-SBI-MS (Naseralavi et. al., 2010) and BP-CGA (Gerist et. al., 2012) using the modal responses of the structure. Only the data of the first three seconds of the earthquake is used.

A 15-ELEMENT CANTILEVER BEAM

A fifteen-element cantilever beam is considered to illustrate efficiency of the proposed method. It has been previously studied by Koh and Dyke (2007). The geometrical and physical data are as follows: the length of beam is 2.74 m; the modulus of elasticity is $2\times10^{11}\ \mathrm{N}/m^2$; the thickness and width are 0.00635 m and 0.0760m, respectively. The elements are numbered from the fixed end as shown in Fig. 1.

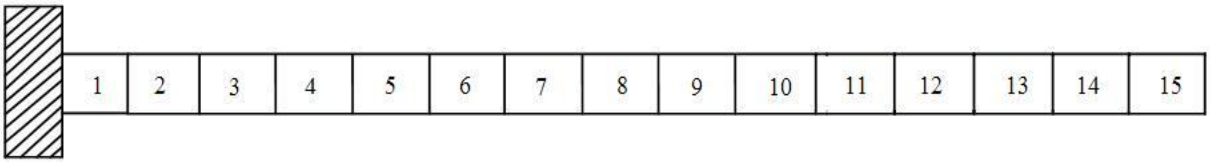
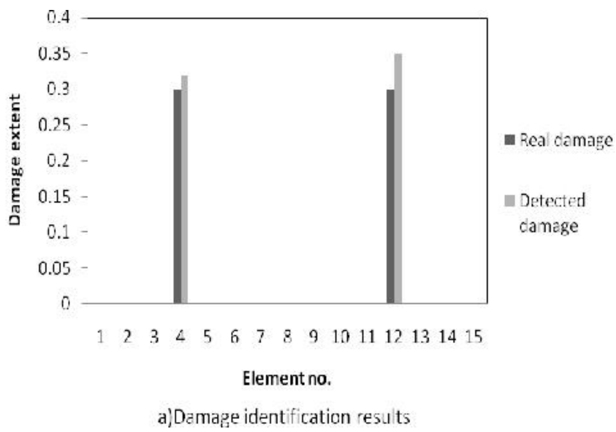


Figure 1. A 15-element cantilever beam

The 4th and 12th elements of the cantilever beam have been assumed to be damaged by the extent of 30%. The damaged elements are efficiently detected by the proposed method as shown in Fig. 2.



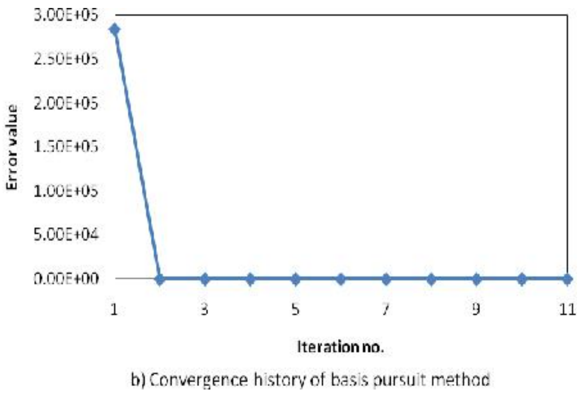


Figure 2. Solution results for the 15-element cantilever beam:
(a) damage identification results (b) convergence history of basis pursuit

The Monte Carlo simulation results of the beam with 10000 times run are shown in Fig. 3. The damage extents of the elements affect the results of the BP method and the error percents increase by increasing the damage extents by BP method. The error percents of BP method are considerably less than the Pseudo-inverse method (PI) for low number of damage elements.

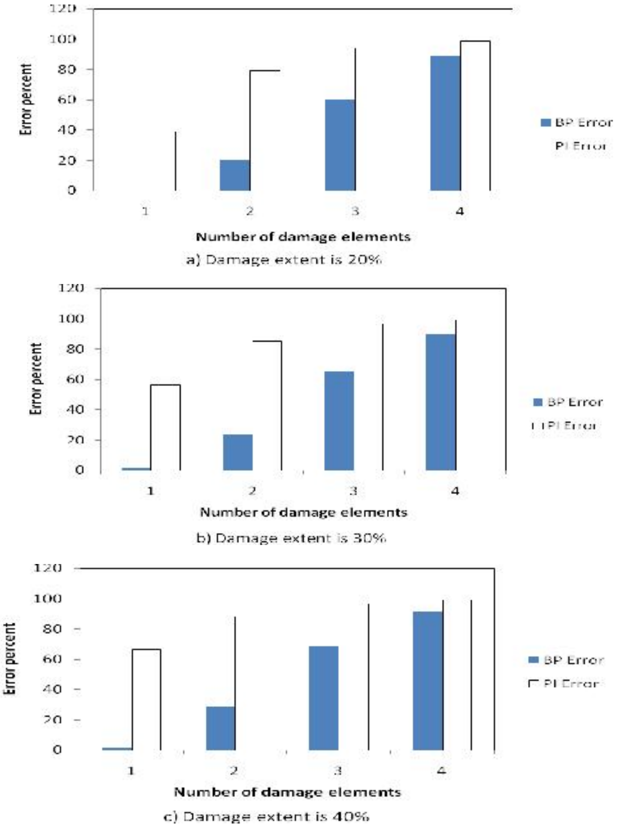


Figure 3. Monte Carlo results of Basis Pursuit (BP) and Pseudo-Inverse (PI) methods for 15-element beam: (a) damage extent is 20% (b) damage extent is 30% (c) damage extent is 40%

Also, run time of the proposed method is significantly less than CGA-SBI-MS and BP-CGA as shown in Table 1.

Table 1. The run time of 15-element cantilever beam

	First stage time (sec)	Second stage time (sec)	Total time (sec)
BP-CGA	2.25	4.2	6.45
CGA-SBI-MS	-	-	17.25
Proposed method	-	-	5.14

A 31-ELEMENT PLANAR TRUSS

A 31-bar planar truss which has been studied by Messina (1998) is selected to demonstrate the capability of the proposed algorithm, as shown in Fig. 4.

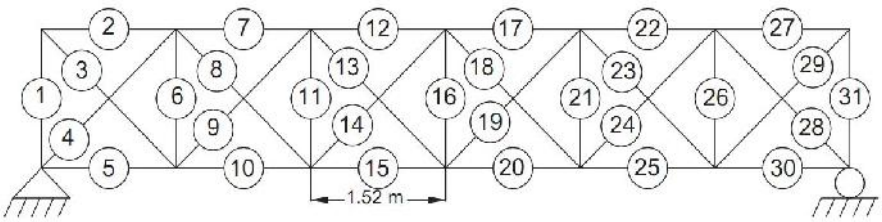


Figure 4. A 31-bar planar truss

The 11th and 25th elements of the planar truss are considered to be damaged by the extent of 50%. The process results are shown in Fig. 5. As it can be seen, the proposed method detects the damage elements efficiently.

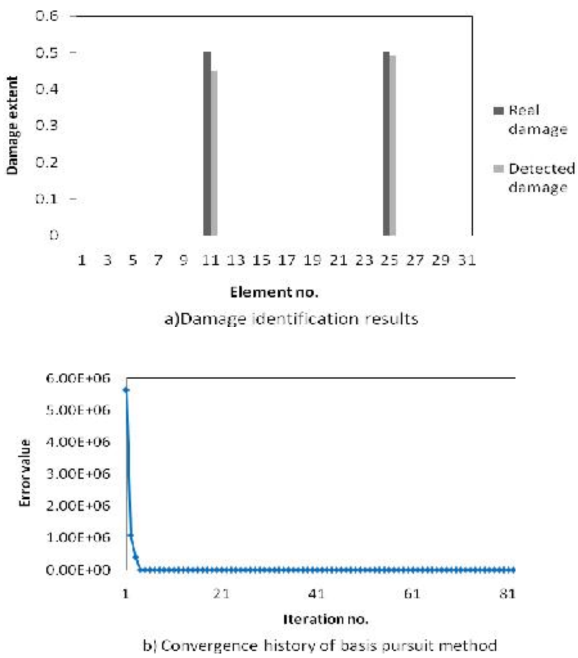


Figure 5. Solution results for the 31-element planar truss:
(a) damage identification results (b) convergence history of basis pursuit

The Monte Carlo simulation results of the truss with 10000 times run are shown in Fig. 6. The results are approximately the same as the beam and BP method significantly has less error than PI.

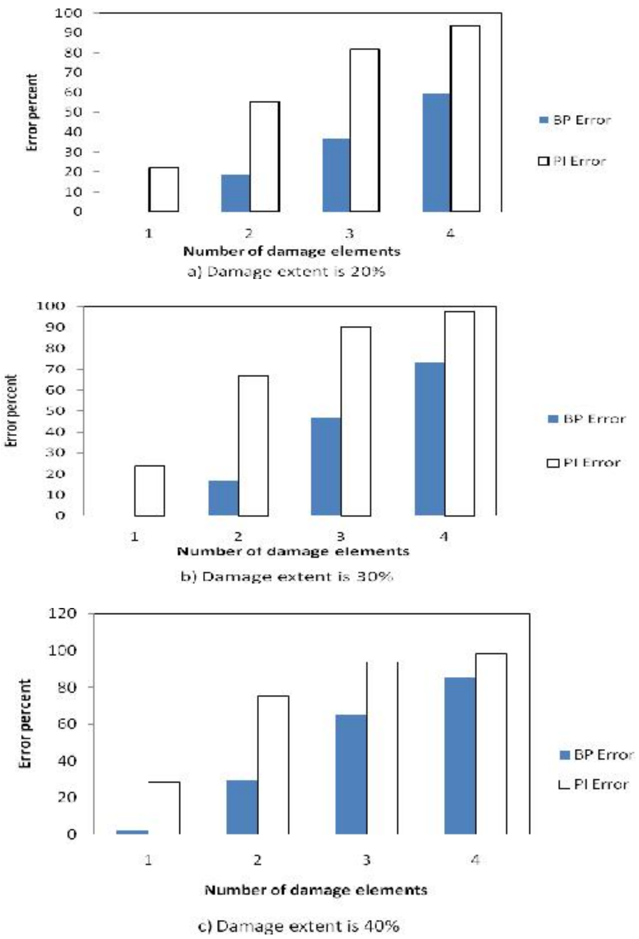


Figure 6. Monte Carlo results of Basis Pursuit (BP) and Pseudo-Inverse (PI) methods for 31-element planar truss:
(a) damage extent is 20% (b) damage extent is 30% (c) damage extent is 40%

Also, run time of the proposed method is significantly less than CGA-SBI-MS and BP-CGA as shown in Table 2.

Table 2. The run time of 31-element planer truss

	First stage time (sec)	Second stage time (sec)	Total time (sec)
BP-CGA	6.46	7.03	13.49
CGA-SBI-MS	-	-	43.04
Proposed method	-	-	4.68

CONCLUSIONS

Damage detection problems are equivalent to a system of equations which relates the damage extents to the structural responses. To solve the system of equations and detect the damage extents and locations, a new method is presented using Basis Pursuit (BP). Basis pursuit method solves the sparse problems using linear programming. Two numerical examples are simulated to detect the damages by the proposed method: a 15element cantilever element and a 1element planar truss. The time history responses of the structures are used to detect the damages. The proposed method efficiently detects the damage locations and extents. This method subsequently requires less time to detect in compare with BP-CGA and CGA-SBI-MS. The efficiency of the proposed method is investigated using Monte Carlo simulation and the results are compared with Pseudo-inverse method (PI). The damage extents of the elements affect the results of the BP method and the error percents increase by increasing the damage extents by BP method. The error percents of BP method are considerably less than PI for low number of damage elements.

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