HOW TO COMPARE THE SEISMIC PERFORMANCE OF STRUCTURES

DimitriosVAMVATSIKOS
Lecturer, National Technical University, Athens, Greece
divamva@central.ntua.gr

Keywords: Seismic Performance, Nonlinear Analysis, Static, Dynamic, Capacity

ABSTRACT

The process of structural design is based on the selection of the top alternative designs from a group of viable choices, ideally choosing the one that best satisfies the requirements. With the emergence of performance-based earthquake engineering, such comparisons now need to be performed on the basis of the seismic performance, preferably at several limit-states. Such a direct evaluation can become cumbersome, requiring seismic hazard information. Therefore, shortcuts and simpler techniques have been introduced that are generally based on the concept of system fragility, as estimated through the various methods of structural analysis. Still, there is no general consensus on the metrics that can be used for such an evaluation. To help with such assessments, we offer a discussion of the available choices for analysts that can employ nonlinear dynamic analysis methods. It is shown that having the complete description of limit-state fragility is sufficient for a reliable comparison, with few exceptions, while lesser information may lead to erroneous results.

INTRODUCTION

The performance comparison of different structural designs, be they alternate structural configurations or simply differently proportioned versions of the same type, is a common, yet little-understood operation in earthquake structural engineering, both in practice and in research. Although it may not explicitly appear in typical engineering calculations,it is a fundamental task that every professional engineer sooner or later encounters. It is essentially the basic premise of seismic design, needed to rationally select, e.g., one structural system or rehabilitation strategy over another, especially when little relevant experience is available.

Limiting ourselves to a given limit-state, the mean annual frequency (MAF) of violating the limit-state is a very powerful way to achieve a robust comparison. Still, there are significant disadvantages that may preclude the wider use of such a method, the most important being the need for seismic hazard information. While it is conceivable, many people would not agree that a comparison of two different designs might shift one way or another based on the site characteristics. It may happen for two relatively close candidates when comparing them at two very different sites, but that is probably not the issue that will trouble most comparisons. Therefore the logical question arrives of whether we can drop the hazard info and focus just on fragility-level information.

This is further motivated by a simplification of the integrals to estimate the MAF into the closed-form solutions developed for SAC/FEMA by Cornell et al. (2002). Thus, if $H(s)$ is the hazard function of the scalar IM represented by variable s, then it can be approximated as

$$H(s) \cong k_0 (s)^{-k} \tag{1}$$

where k is a positive constant, indicative of the local hazard slope at the range of interest. We need also assume that the relationship of IM and EDP, the latter represented by the variableθ, is approximately a power law:

$$s \cong a \cdot \bar{\ }^{\,b} \tag{2}$$

Then, if $\hat{\bar{\ }}_c$ is the median EDP capacity and $\beta_{T\theta}$ is the total dispersion (standard deviation of the logarithm of the data) in EDP demand and capacity due to both aleatory randomness and epistemic uncertainty, the MAF of LS can be estimated as

$$\mathrm{a}_{\mathrm{LS}} \cong H\left[\left(\frac{\hat{\bar{\ }}_c}{a}\right)^{\frac{1}{b}}\right]\exp\left(\frac{k^2}{2b^2}\ \ {}^{2}_{T^{-}}\right) \tag{3}$$

Similarly, by employing only the hazard curve approximation of Eq. (1), if $\hat{s}_c$ is the median IM capacity, and β_{TSc} is the corresponding total dispersion, Eq.(3) turns to

$$\mathrm{a}_{\mathrm{LS}} \cong H(\hat{s}_c)\exp\left(\frac{k^2}{2}\ \ {}^{2}_{TSc}\right) \tag{4}$$

Thus, a MAF-based comparison may be easily performed by using the approximate SAC/FEMA closed-form solutions. Still, some care needs to be exercised when doing so because of the assumptions included. Some, like homoscedasticity and lognormality of the IM and EDP capacities are easy to satisfy in the area of interest. On the other hand, the functional approximations employed are more difficult to justify. These closed-form solutions are sensitive to the way that the fitting is performed for deriving the constants in the power law approximations of Eq. (1) and (2). Especially regarding the hazard curve, small changes in the capacities may produce disproportionately large changes in the MAF estimated, especially if a tangent fit is performed at the median IM capacity, as proposed in Cornell et al (2002). A biased local fit at IM values lower than the median capacity will in general improve the robustness of the fit (see Dolsek and Fajfar 2008, Vamvatsikos 2014).

In general, if we trust the approximation of Eq. (3), (4), then it all depends on the few variables present in the equations, most notably the median IM or EDP capacities, the associated dispersions and the constants k and b related to the hazard curve and the IM–EDP relationship. In other words, it all depends on the distributions of capacity and one or two constants depending on whether we employ Eq. (4) or (3), respectively. Assuming that the two candidate designs allow us to use the same intensity measure (e.g. the spectral acceleration,S_a, on the same period) we can envision making our comparisons directly on the IM–EDP plane.

Considering that incremental dynamic analysis (IDA,Vamvatsikos and Cornell 2002) is arguably the prime method for establishing in detail the complex IM–EDP relationship for any given structure, we will call such a methodology a comparison in IDA-space, as envisioned in Fig. 1. Invariably, this should serve as a reminder that despite the simplifications achieved coming down from the top of the comparison pyramid that has been established, such a comparison may still entail numerous nonlinear dynamic analyses under a multiply-scaled suite of ground motion records. Simpler methods do exist that based on Eq. (3) are able to approximate the limit-state MAF without using full IDA. Instead, they employ only a few nonlinear dynamic analyses at one or two levels (stripes) of a given IM(Jalayer and Cornell 2009) but at the heavy cost of reduced accuracy. Therefore, if uncompromised resolving power is what we are after, IDA or an equivalent multi-stripe analysis (Jalayer and Cornell 2009) is a one-way street.

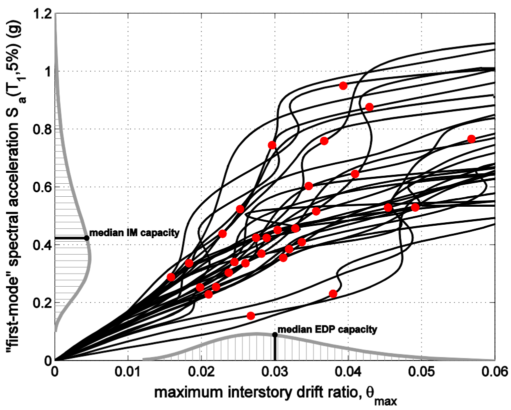

Figure 1. Thirty IDA curves, thirty limit-state capacity points and the corresponding probability density functions for the EDP and IMcapacities of a 9-story steel frame (from Vamvatsikos 2013)

IM CAPACITY BASIS

In general, comparing in terms of the capacity in terms of anIM is a multi-faceted issue. First of all, while recent work has shown that vector or composite IMs can be very useful for reasons of efficiency and sufficiency (Baker and Cornell 2005, Vamvatsikos and Cornell 2005, Luco and Cornell 2007) such complex IMsmay be problematic for our purposes. For example, when dealing with vectors, they cannot be compared by definition. Thus, options like $S_a(T_1)$ and epsilon (Baker and Cornell 2005) or a set of two S_a'sat different periods (Vamvatsikos and Cornell 2005) become impossible to use. In addition, we cannot easily compare structure-dependent IMs, like the inelastic spectral displacement $S_{di}(T_1)$ (Tothong and Cornell 2008), since it heavily depends on the equivalent single-degree-of-freedom (SDOF) force-deformation relationship that is building specific. Actually, even the simple $S_a(T_1)$ is structure-dependent and needs a common period selection, rather than the actual first-mode period of each structure, to become a standard for comparison.

Perhaps one possibility is to fall back to the use of peak ground acceleration (PGA) or velocity (PGV). While this may seem as a good idea it can easily remove sufficiency (Luco and Cornell 2007) and thus diminishthe accuracy in our approach on the basis of invalidating even a MAF approach on such an IM for many structures. As a balance between the difficult requirements of comparability and sufficiency (i.e., ease versus validity of the comparison) we propose the use of $S_a(T)$ that can be made to work if we shift both structures to a common period T that lies between their actual fundamental periods (Fragiadakis et al. 2006)]. Of course it would be best if we avoid large period changes, as other sufficiency questions will again come into play. Barring such issues, any IM-capacity-basis comparison is essentially a fragility-based comparison, and can be easily resolved by comparing the cumulative distribution fucntions (CDFs) of capacity.

On the premise of a common IM description, we can easily compare the performance of structures 1 and 2 at any given limit-state LS. Since the MAF is estimated as the integral of the fragility times the hazard slope a simple mathematical argument can be made: If there is a large difference between the median $s_c$values of the two structures, say $\hat{s}_{c1} > \hat{s}_{c2}$ and assuming that the dispersion β_{TSc1} of the first is not extremely larger than β_{TSc2}, or in other words the CDF of s_{c1} is nearly always to the right of the CDF of s_{c2}, then it is quite obvious that due to the monotonically decreasing nature of the hazard curve (and the monotonically increasing nature of its absolute derivative), the exceedance probabilities of s_{c2} will always be multiplied by higher slope values and always lead to higher MAFs. Therefore, in this situation the inequality of the median capacities directly translates to an inverse inequality of the corresponding MAFs.If there is only a small difference between the median capacities of the two structures, then the above argument cannot be used any more. Nevertheless, we can now take advantage of the approximate Eq. (4), as the proximity of the capacities allows the same hazard curve approximation to be employed. Since there is now a common description of the hazard via Eq. (1) for both structures, the following result is easily derived:

$$a_{LS1} < a_{LS2} \Leftrightarrow \hat{s}_{c1} > \hat{s}_{c2} \cdot \exp\left[\frac{k}{2}\left({}^{2}_{TSc1} - {}^{2}_{TSc2} \right)\right] \quad (5)$$

Therefore the hazard curve may influence the results only if the dispersions are appreciably different. If testing for structure 1 being better than structure 2, as implied by the above equation, then the higher the dispersion of the capacity in structure 2 relatively to 1, the easier it is to satisfy the inequality we are seeking. If, on the other hand, the dispersions are relatively close together, say within 5-8%(e.g. 0.45 versus 0.4), then the influence of the hazard disappears for all practical purposes. Higher dispersions for the high-capacity structure actually heavily favor the seemingly inferior structure, as they disproportionately increase the probability of failure for the former. In that case, a fragility-based comparison may fail.

Summing up, the only doubt cast on a pure IM-based comparison appears when one of the structures combines a slightly higher median capacity (say up to 20% higher) with significantly higher capacity dispersion than its competition. Then, one needs to resort back to MAFs to make sense of the comparison. Otherwise, the inequality of the median capacities is directly linked to a performance inequality. Since relatively similar structural configurations at nearby periods are generally expected to have very similar dispersions, there is every reason to support the validity of an IM-basis for comparisons.

This approach is best exemplified by the work of Fragiadakis et al. (2006) where the ratios of the median IM-capacities were used to qualitatively and quantitatively measure the influence of mass, strength and stiffness irregularities along the height of a 9-story steel frame. Their application also included the use of bootstrapped confidence intervals (Fig. 2) on the capacity ratios to take into account the influence of sample size, i.e., of the limited number of ground motion records employed for IDA. While this factor often tends to be neglected, its inclusion is heavily recommended as many close comparisons are often rendered completely inconclusive due to sample size effects.

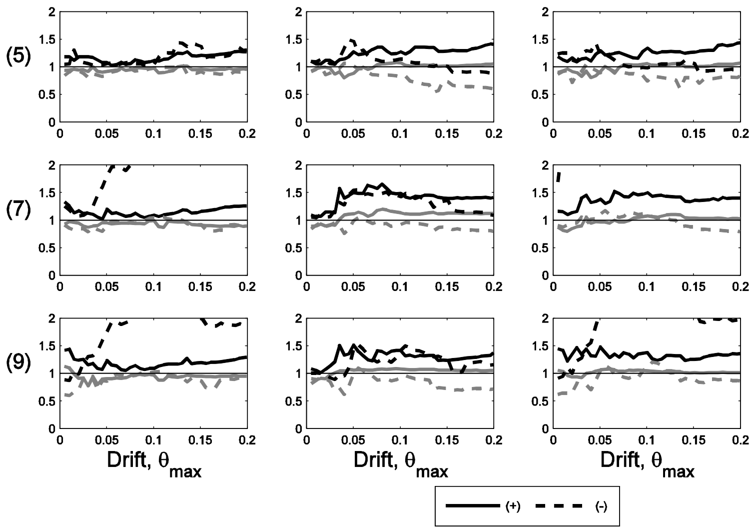

Figure 2. Changes in stiffness (left), strength (center), or both (right) for stories 5, 7 and 9 of a 9-story steel moment-resisting frame. Light gray lines indicate the lower bound and darker ones the upper bound of the 90% confidence interval (from Fragiadakis et al. 2006)

EDP CAPACITY BASIS

Perhaps the most important problem with an IM comparison is the apparent difficulty in obtaining the distribution parameters on a “required IM” basis. From one point this necessitates the use of some clever postprocessing (Vamvatsikos and Cornell 2004) to extract the needed information. It may also be argued that the concept of an IM capacity is still foreign to engineers, while an EDP capacity is far more intuitive. It thus makes sense to ask whether it would be possible to make comparisons on the basis of the median EDP capacity. The concept of the EDP capacity was introduced through SAC/FEMA (2000) as a convenient way to represent the capacity of structures via Eq. (3). Through this closed-form solution it is obvious that EDP capacity is related to the MAF, albeit needing one more constant than an IM basis, namely the parameter *b* that relates to the shape of the median IM–EDP relationship. Unfortunately there are many problems plaguing such a comparison than just this constant.

One major issue in this approach is the definition of the EDP capacity itself. Even according to SAC/FEMA (2000), for most limit-states the EDP capacities are essentially fixed values, same for a whole class of structures. For example, for an Immediate Occupancy limit-state there is a uniform 2% maximum interstory drift capacity for all steel moment-resisting frames. Similarly, more recent guidelines specify fixed beam or column plastic rotation capacities that only differ among different classes of buildings, typically based on a ductility classification, e.g. Eurocode 8 (CEN 2005). Clearly no comparison can be made on the basis of such constant-value definitions. Still, it may seem attractive to use them if they vary from building to building.

There are indeed cases where building-specific EDP capacities have been defined in a way that is characteristic of the performance of structures. That is the case of the SAC/FEMA Collapse Prevention (CP) limit-state, where the “drift capacity” is defined as the point where the IDA curve softens to less than 20% of its initial elastic slope with a maximum allowable capacity value of 0.10. On this basis, capacity-points can be defined either on each IDA curve individually, as done in Fig. 1for thirty records, or on the median curve itself, as shown in Fig. 3 for two idealized median curves. The differences between the two modes of application are insignificant and the final results are quite similar in both cases. Despite the various issues with such a definition itself (Vamvatsikos and Cornell 2004), it may nevertheless open a door for a useful, period-free measure of performance that can be used for comparison. Actually, although not explicitly used for this purpose, such values of drift capacities are catalogued in the work of Liao et al. (2007) and Huang and Foutch (2009), where they may be assumed to be indicative of the collapse potential of different structural systems or configurations of the same system.

Unfortunately, as Fig. 3 shows, an EDP-capacity basis for comparison may prove to be misleading. Therein we show the idealized median IDA curves of two different systems that have the same period and initial elastic slope but quite different flatlines, i.e., different IM-levels where global dynamic instability manifests itself. These IDA curves are shown to have slight differences in the curvature as they rise towards their flatlines that force the EDP capacity of system 1 to be lower than that of 2, although the relation of their median IM capacities is reversed. Therefore, based on our previous discussion on IM-comparisons, system 1 will clearly outperform system 2. A comparison based on their EDP capacities would erroneously indicate otherwise. In other words, using this simple counter-example, we can show that EDP-based comparisons can be severely flawed.

Let us now seek the theoretical reason behind this discrepancy. Following the same route we took to establish the validity of the IM-comparison, we can assume that as shown in Fig. 3 the systems have the same period and their capacities are close enough to allow us using the same hazard curve approximation via Eq. (1). Then, using the SAC/FEMA approximation of Eq. (3), and if we let a_i, b_i (i= 1,2) represent the IDA-shape parameters of Eq. (2) for each of the two systems, we arrive at the following requirement:

$$\mathrm{a}_{\mathrm{LS1}} < \mathrm{a}_{\mathrm{LS2}} \Leftrightarrow \hat{\bar{\ }}_{c,1} > \hat{\bar{\ }}_{c,2} \cdot \left(\frac{a_1}{a_2}\right)\exp\left[\frac{k}{2}\left(\frac{\bar{T}_1^{2}}{b_1} - \frac{\bar{T}_2^{2}}{b_2}\right)\right] \qquad (65)$$

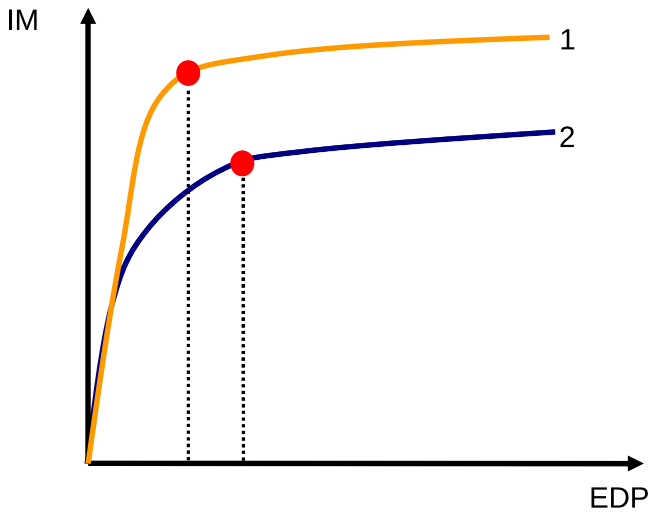

Figure 3. Two idealized median IDAs with different IM and EDP capacities

The above relation clearly shows that the outcome of the comparison does not depend only on the EDP capacities but also on the parameters that represent the IDA curve. In other words, the shape implied by a_i, b_i is a very important consideration that, as exemplified by Fig. 3, cannot be discarded, even if we assume the very simple power-law shape implied by Eq. (2). Furthermore, Eq. (3), and consequently Eq. (6), has another important limitation: it is not able to take into account the probability of collapse which may become important when close to the flatline. For all of the above reasons, unless under quite restrictive assumptions, any comparison on an EDP basis should not be considered indicative of the actual performance and it should be avoided in general.

EDP DEMAND BASIS

Another intuitive way of comparing two different structures involves performing a single stripe analysis, for example at the design level IM corresponding to, e.g., 10% in 50 yrs for a Life Safety comparison, and comparing the statistics of the EDP response of structure 1 versus those of structure 2. This does away with the need for a cumbersome IDA that the previous approaches dictate, restricting the computational complexity to a single intensity level rather than multiple ones, making this a very attractive proposition.While such a comparison may seem simple, especially if the dispersions are similar where it would point towards a comparison of mean or median EDP responses, it only provides some evidence that may inconclusively favor one of the two structures. It does not allow for a definitive comparison unless the difference of the responses is disproportionately large and some severe constraints and assumptions are in place. There are two important reasons for this.

One problem is the issue of collapse, i.e. some ground motions may be found to produce "infinite" EDP results due to non-convergence of the dynamic analysis that signals, on a well-executed analysis and a numerically robust model, the onset of global dynamic instability. Then, the envisioned "simple" comparison of two mean or median EDP responses may become a difficult multi-criteria approach: Structure 1 may show a 3% drift and 20% probability of collapse (e.g. 4 out of 20 records have not converged), while structure 2 has 3.5% drift and 10% probability of collapse. Which one is preferable? One cannot answer such a question without resorting to an IM-based technique. Therefore, when the probability of collapse cannot be ignored, such a single-IM-level comparison may not even be feasible.

The most important reason, though, is that information at only a single level of intensity cannot provide much intuition on what is happening at other intensities. There is indeed some correlation to be expected among responses at nearby intensities but this does not change the fact that our information is restricted to a point-estimate of the relationship between the two structures. As even the simple Eq. (3) shows, that is not enough to allow us to determine the performance even for a single limit-state without some information about the shape of the complete IDA curves. It is easier to understand the problem if we realize that we cannot determine the probability of exceedance of a given EDP-limit only by testing at a single level of intensity, simply because of the record-to-record variability. Due to such variability there are both lower and higher IM levels than the one tested that may cause high enough EDP response that will significantly contribute to the exceedance of the limit-state. Actually, considering that the lower IMs are associated with disproportionately higher exceedance frequencies, they often contribute much more to the overall performance. In other words, the comparative shapes of the IDA curves, or, as discussed in the previous section, the terms a_i and b_i, remain equally important as before and it is not possible to remove them from the comparison. Unless we can make certain assumptions about their comparative values, which can be quite hazardous in many cases, we should refrain from using such narrow-range comparisons.

CONCLUSIONS

Comparing design alternatives on the basis of performance is not straightforward, as there are a lot of metrics, each with its own implementation issues and sometimes plagued by many pitfalls and fallacies. Generally, it can be said that top level, holistic approaches such as using an all-encompassing lifecycle cost or even annualized losses in terms of one or more decision variables like repair cost, downtime or casualties, is the only sure way. While comprehensive, such approaches are very cumbersome and still remain impractical for most situations.

Thus it becomes a much needed simplification to move to an engineer-friendly structural limit-state basis, where all seismic hazard, cost and damage information is removed to leave only a clean fragility-based comparison. Such an approach can take us back to the familiar ground of seismic intensity versus structural response and it comes with many benefits but also some costs. The best compromise is using a common intensity measure, for example the spectral acceleration at a common period, to allow a comparison in terms of the median IM-capacity. This needs IDA-level information, i.e., multiple nonlinear dynamic analyses under multiple earthquake records spanning different intensity levels, but retains all the robustness of a performance basis by being directly linked to the mean annual frequency of exceeding a limit-state. The seemingly attractive approach of using a common EDP as the basis of comparison, either in the form of EDP capacity or as statistics of EDP response at a single intensity level, should be avoided as it tends to be unreliable.

REFERENCES

Baker JW and Cornell CA (2005) A vector-valued ground motion intensity measure consisting of spectral acceleration and epsilon, *Earthquake Engineering and Structural Dynamics*,34:1193–1217

CEN (2005) Eurocode 8: Design of structures for earthquake resistance, EN 1998-3, European Committee for Standardisation, Brussels

Cornell CA, Jalayer F, Hamburger RO andFoutch DA (2002) Probabilistic basis for 2000 SAC Federal Emergency Management Agency Steel Moment Frame Guidelines,*Journal of Structural Engineering ASCE*, 128(4):526-533

Dolsek M and Fajfar P (2008) The effect of masonry infills on the seismic response of a four storey reinforced concrete frame–a probabilistic assessment,*Engineering Structures*,30:3186–3192

Fragiadakis M, Vamvatsikos D andPapadrakakis M (2006) Evaluation of the influence of vertical irregularities on the seismic performance of a 9-storey steel frame, *Earthquake Engineering and Structural Dynamics*,35(12):1489–1509

HuangZ and FoutchDA (2009) Effect of hysteresis type on drift limit for global collapse of moment frame structures under seismic loads,*Journal of Earthquake Engineering*, 13:939-964

Jalayer F and Cornell CA (2009) Alternative non-linear demand estimation methods for probability-based seismic assessments,*Earthquake Engineering and Structural Dynamics*, 38(8):951-1052

Liao K-W, Wen Y-K andFoutch DA (2007) Evaluation of 3D steel moment frames under earthquake excitations I: Modeling,*Journal of Structural EngineeringASCE*, 133(3):462–470

Luco N and Cornell CA (2007) Structure-specific scalar intensity measures for near-source and ordinary earthquake ground motions, Earthquake Spectra, 23(2):357–392

SAC/FEMA (2000)Recommended seismic design criteria for new steel moment-frame buildings,Report No. FEMA-350,prepared for the Federal Emergency Management Agency,Washington DC

Tothong P and Cornell CA (2008) Structural performance assessment under near-source pulse-like ground motions using advanced ground motion intensity measures, *Earthquake Engineering and Structural Dynamics*,37:1013–1037

Vamvatsikos D (2013) Derivation of new SAC/FEMA performance evaluation solutions with second-order hazard approximation,*Earthquake Engineering and Structural Dynamics*, 42:1171–1188

Vamvatsikos D (2014)Accurate application and second-order improvement of SAC/FEMA probabilistic formats for seismic performance assessment,*Journal of Structural EngineeringASCE*, 140(2),04013058

Vamvatsikos D and Cornell CA (2002) Incremental dynamic analysis,*Earthquake Engineering and Structural Dynamics*,31(3):491–514

Vamvatsikos D and Cornell CA (2005) Developing efficient scalar and vector intensity measures for IDA capacity estimation by incorporating elastic spectral shape information, *Earthquake Engineering and Structural Dynamics*, 34(13):1573–1600

Vamvatsikos D and Cornell CA (2004) Applied Incremental Dynamic Analysis,*Earthquake Spectra*,20(2):523-553