THE PSEUDO-DYNAMIC ANALYSIS OF CRITICAL NON-CIRCULAR SLIP SURFACE IN EARTH SLOPES

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ABSTRACT

This paper presents a new approach to determine the minimum factor of safety and the associated critical line segment pseudo-dynamic slip surface. The new technique is called Alternating Variable Local Gradient (AVLG) that it is based on Univariat method. The slip surface shape is line segment (or non-circular). The safety factor is calculated using Janbu function. This procedure can truly convert the initial slip surface to critical slip surface with the different conditions of slope and soil properties. The critical slip surface obtained in this method provides both minimum factor of safety and more accordance with reality in nature. Resultant critical slip surface is reasonable also comparative study of numerical examples indicate the efficiency and accuracy of proposed method.

INTRODUCTION

Evaluating the stability of slopes in soil is an important, interesting and challenging aspect of civil engineering. Over the past decades, the calculation of the factor of safety and the search for a slip surface with the critical or minimum factor of safety, has led to development of new and more effective methods. In most of the commercial softwares, the search for critical circular slip surface is available. Due to the limitation of the commonly used slope stability analysis programs which cannot locate the critical non-circular failure surface of a slope under general conditions with general constraints, most of the engineers are forced to perform the search on a trial and error basis (Cheng, 2003). In first works on optimization of slope stability various simple methods have been used by different researchers. Celestino and Duncan (Celectino and Duncan, 1981) described a simple search method, referred to as the alternating variable technique to determine critical noncircular slip surfaces. This simple numerical method, applicable to practical problems involving layered soils and other complexities, is used to search for the critical slip surface. To search for the non-circular surface with the lowest factor of safety, a number of repeated trials are performed. Each trial begins by shifting each of the points defining the slip surface to two new positions, and calculating the factor of safety. As this is done, all the other points are kept at their original estimated positions. Furthermore Nguyen (Nguyen, 1985) presented on optimization technique to determine both the minimal factor of safety and the associated critical failure surface. The technique treats the factor of safety as an optimal function of the N geometrical coordinated defining potential admissible failure surfaces, and searches for the optimum by a simplex reflection algorithm (in conjunction with the Omit equilibrium) through an N-dimensional space defined by these geometrical coordinates. On the other, the optimization techniques for the slope stability have also been used using variational limiting equilibrium by different researchers. Leshchinsky (Leshchinsky, 1990) presented a generalized slope stability method, which is based on the variational limiting equilibrium analysis introduced by Baker and Garber (Baker and Garber, 1977).

The differential equation describing the normal stress distribution is determined analytically through a process of minimization of the factor of safety while, simultaneously, satisfying all the three limiting equilibrium equations for the entire sliding body. Jiang and Magran (Jiang and Magran, 1997) described a new finite element method of limit analysis to determine automatically the optimal failure mechanism for geometrically complicated structures under applied loads, or limited load. A new finite element method of limit analysis does not need the engineer to assume the failure mechanism because it can automatically find the critical failure mechanism which gives the least resistance of the structure to the applied loads. Arai and Tagyo (Arai and Tagyo, 1985) proposed a numerical procedure to determine the location of the critical non-circular slip surface giving the minimum factor of safety. The slope subjected to analysis is divided into slices, each of which has a finite width. They employed the simplified Janbu's method as the tool of stability analysis. Using the optimization technique, the procedure was capable of step-wisely correcting the location of slip surface until the factor of safety reduced/converged to a sufficiently small value. Bellezza (Bellezza, 2004) presented the optimization of landfill volume by the simplex method, and the proposed approach can be a useful tool in the landfill design and in the evaluation of landfill capacity. The main limitation of the proposed approach is that the 3D slip surfaces are defined a priori, and therefore, they are not necessarily the most critical. Cheng (Cheng, 2003) presented on the treatment of constraints, simulated annealing technique for location of critical circular and non-circular failure surface under general conditions. One of the earliest procedures of analysis for seismic stability is the pseudostatic procedure, in which the earthquake loading is represented by a static force, equal to the soil weight multiplied by a seismic coefficient, k. The pseudostatic force is used in a conventional limit equilibrium slope stability analysis. The seismic coefficient may be thought of loosely as an acceleration that is produced by the earthquake. Application of a seismic coefficient and psedustatic force in limit equilibrium slope stability analysis is relatively straightforward from the perspective of the mechanics: The pseudostatic force is assumed to be a known force and is included in the various equilibrium equations (Duncan and Wright, 2005). This paper presents AVLG technique for pseudostatic analysis of critical line segment slip surface in earth slopes.

THE LIMIT EQUILIBRIUM METHOD

All procedures of slices as well as the single free-body procedures are based on the relatively simple shape for the slip surface: a plane, a logarithmic spiral, or a circle. Many times the slip surface is more complex, often following zones or layers of relatively weak soil of weak interfaces between soil and other materials, such as geo-synthetics. In such cases, it is necessary to compute stability using more complex shapes for the slip surface. Several procedures have been developed for analysis of more complex, non-circular slip surfaces. However, line segment slip is focused on in this study.

In this section presents an equation to calculate factor of safety based on limit equilibrium method which includes the seismic coefficient and reinforcement forces. Procedure based on line segment slip surface consider equilibrium of forces in x and y directions and also equilibrium of moment about the toe of the slope. Figure 1 illustrates a slope and a slice with seismic (*kw_i*) and reinforcement (*T_i*) forces.

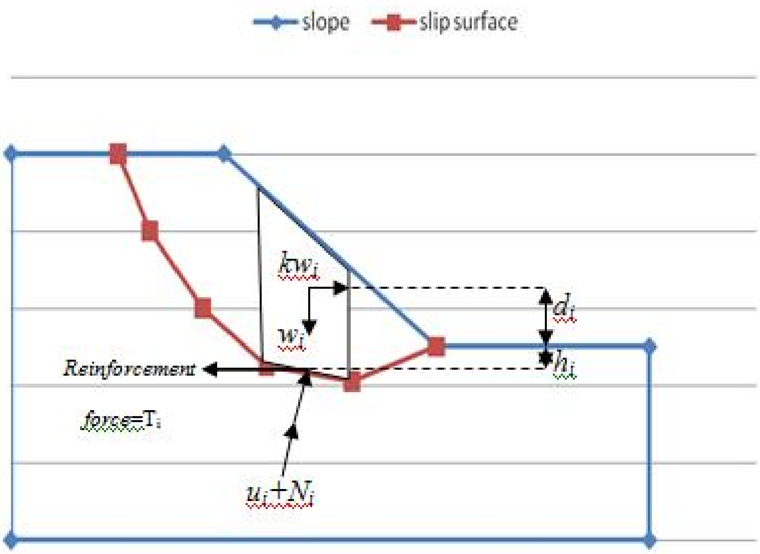

Figure 1. Seismic and reinforcement forces in a slice

Referring to the slice shown in Figure 1 and equilibrium of moment about the toe of slope we can write:

$$\sum \frac{s_i \Delta l_i}{FS} \cos\alpha_i h_i - \sum w_i \sin\alpha_i \cos\alpha_i h_i - \sum k w_i d_i + \sum T_i \cos\psi_i h_i = 0 \qquad (1)$$

where s_i= shear strength; Δl_i= thickness of a slice; α_i= angle of inclination of the slip surface for ***i-th*** slice; w_i= the weight of the ***i-th*** slice; k= seismic coefficient; T_i= reinforcement force; ψ_i= angle of reinforcement force is inclined from the horizontal.

The equation for the factor of safety that satisfies moment equilibrium then becomes:

$$FS = \frac{\sum [c' + (\sigma - u)\tan\phi'] h \Delta l \cos\alpha}{\sum h w \sin\alpha \cos\alpha - M_n} \qquad (2)$$

$$M_n = \sum T_i \cos\psi_i h_i - \sum k w_i d_i \qquad (3)$$

where c'=effective cohesion; σ =normal stress; u=pore water pressure.

Resolving forces in the vertical direction, the following equilibrium equation can be written for forces in the vertical direction:

$$N \cos\alpha + S \sin\alpha - w + T \sin\psi = 0 \qquad (4)$$

For shear strengths expressed in terms of effective stresses with Mohr-Coulomb strength equation, we can write:

$$S = \frac{1}{FS}[c'\Delta l + (N - u\Delta l)\tan\phi'] \qquad (5)$$

Combining Eqs. 4 and 5 and solving for the normal force, N, we obtain:

$$N = \frac{w - T\sin\psi - \frac{1}{FS}(c'\Delta l - u\Delta l \tan\phi')\sin\alpha}{\cos\alpha + \frac{1}{FS}\tan\phi' \sin\alpha} \qquad (6)$$

Combining Eq. 6 for normal force with Eq. 2 for the factor of safety then gives:

$$FS = \frac{\sum \dfrac{[(w_i - F_{vi} - u_i \Delta l \cos\alpha_i \tan\phi_i' + c_i' \Delta l \cos\alpha_i]\cos\alpha_i h_i}{\cos\alpha_i + \frac{1}{FS}\tan\phi_i' \sin\alpha_i}}{\sum w_i \sin\alpha_i \cos\alpha_i h_i - M_n} \qquad (7)$$

$$F_{vi} = T_i \sin\psi_i \qquad (8)$$

OPTIMIZATION TECHNIQUE

Optimization is the act of obtaining the best result under given circumstances (Rao, 1991). In design, construction and maintenance of any engineering system, engineers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is to either minimize the effort required or maximize the desired benefit. There are various methods for solving an unconstrained minimization problem, such as Direct Search methods (Hooke and Jeeves, 1961), Random Search method (Rao, 1991), Univariate method (Rao, 1991), Pattern Search methods (Rao, 1991), Rosenbrock’s method (Rao, 1991), Steepest Descent method (Cauchy, 1847), Conjugate Gradient method (Fletcher and Reeves, 1964), Quasi-Newton methods (Rao, 1991) and Variable Metric method (Davidon, 1959; Fletcher, 1970). Through combining these methods, a method has been developed for minimizing factor of safety in slope stability analysis that is called Alternating Variable Local Gradient by the authors. This method is efficient for line segment slip surface in slope stability analysis to obtain the minimum factor of safety. In the AVLG method, first it is necessary to find the critical circular slip surface using Grid search method. In the Grid search method, a number of slip surfaces must be assumed to find the slip surface that produces a minimum factor of safety. The surface with the minimum factor of safety is termed the critical slip surface. Then, we select the number of nodes on the critical circular slip surface as initial line segment (non-circular) slip surface to optimum it, or to find the minimum factor of safety. The gradient is a 2n-component vector, and it has a very important property. The node moves along the gradient negative path in 2D-dimensional space, the function value increases at the fastest rate. In slope stability analysis, after is obtained critical circular slip surface and is selected initial line segment slip surface on the critical circular slip surface, we move an only first boundary node on the boundary to obtain the local minimum factor of safety and other nodes are fixed. Then we move the only next node of line segment slip surface, along the local gradient negative path and other nodes are fixed to obtain the local minimum factor of safety (Figure 2). Then we move another node along the local gradient negative path and other nodes are fixed to obtain the local minimum factor of safety. This approach is repeated for total nodes, until the local minimum factor of safety is found. Note that the boundary nodes are moved only on the boundary. Thus, the final local minimum factor of safety is the global minimum factor of safety, for line segment slip surface.

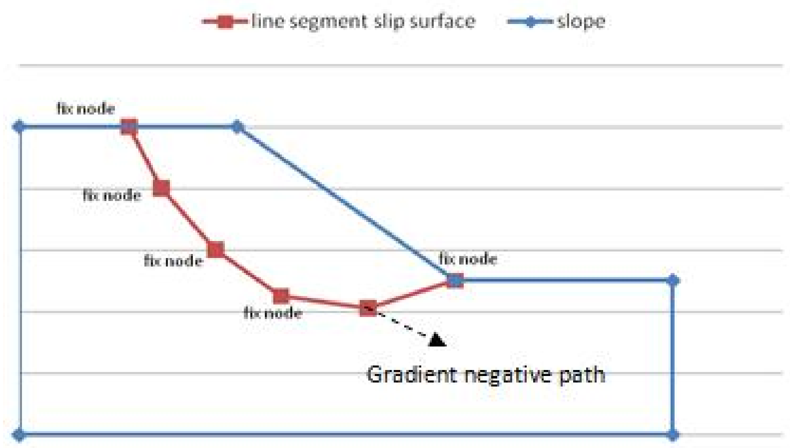

Figure 2. Only one node is moved along the local gradient negative direction and other nodes are fixed

This procedure is given by the following steps that it is based on univariate method (Rao, 1991).

Step 1. finding the critical circular slip surface using Grid search method.

Step 2. selecting the initial line segment slip surface on the critical circular slip surface using number nodes. It is better to select number of nodes between 5 to 15 nodes.

Step 3. selecting the more nodes of slip surface in weak layers for non-homogeneous slope.

Step 4. moving only the first node along the boundary until the local minimum factor of safety is found.

Step 5. moving only the second node, in the direction of the local negative gradient until the local minimum FS is found. The gradient of the function is:

$$\underset{2n\times 1}{\nabla \mathrm{FS}} = [0, 0, \frac{\partial \mathrm{FS}}{\partial x_2}, \frac{\partial \mathrm{FS}}{\partial y_2}, 0, 0, \ldots, 0, 0]^{-1}$$

Step 6. moving only the third node, in the direction of the local negative gradient until the local minimum FS is found. In this step, gradient of the function is:

$$\underset{2n\times 1}{\nabla \mathrm{FS}} = [0, 0, 0, 0, \frac{\partial \mathrm{FS}}{\partial x_3}, \frac{\partial \mathrm{FS}}{\partial y_3}, 0, 0, \ldots, 0, 0]^{-1}$$

Step 7. moving only the (n-1)th node in the direction of the local negative gradient until the local minimum FS is found as,

$$\underset{2n\times 1}{\nabla \mathrm{FS}} = [0, 0, 0, 0, \ldots, \frac{\partial \mathrm{FS}}{\partial x_{n-1}}, \frac{\partial \mathrm{FS}}{\partial y_{n-1}}, 0, 0]^{-1}$$

Step 8. moving only the end node along the boundary until the local minimum factor of safety is found.

Step 9. repeat step 4 to 8 while the difference between FS for two last slip surface to be little than 1e-5.

The latest FS is a global minimum factor of safety. The final location of any node must be the kinematically acceptable shape of failure surface. The process described, is effective and efficient to egress from the local critical line segment slip surface and finding the global critical line segment slip surface.

EXAMPLES

Example1: The first example considers a simple slope of a homogeneous soil with zero pore water pressure. This example has also been considered by Arai and Tagyo (Arai and Tagyo, 1985). The values of the safety factor are determined from Equation 1 and critical slip surface is obtained by AVLG method. The critical circular slip surface is determined using search method and six nodes are selected on initial slip surface (Figure 3).

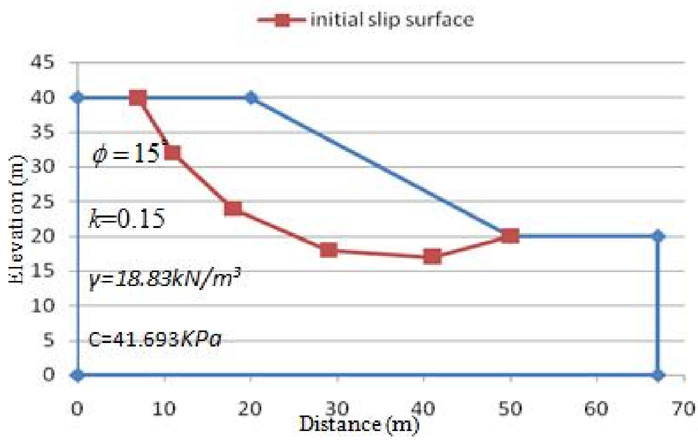

Figure 3. Properties and initial slip surface in example 1

The seismic coefficient is 0.15. The initial slip surface is optimized after 5 trials. Figure 4 shows the critical line segment slip surface. Table 1 demonstrates the results that are obtained by the present procedure. The minimum factor of safety for circular slip surface is 1.322 and minimum factor of safety for line segment (non-circular) with the present procedure is 1.15, which indicates that the factor of safety is improved about 13 percent.

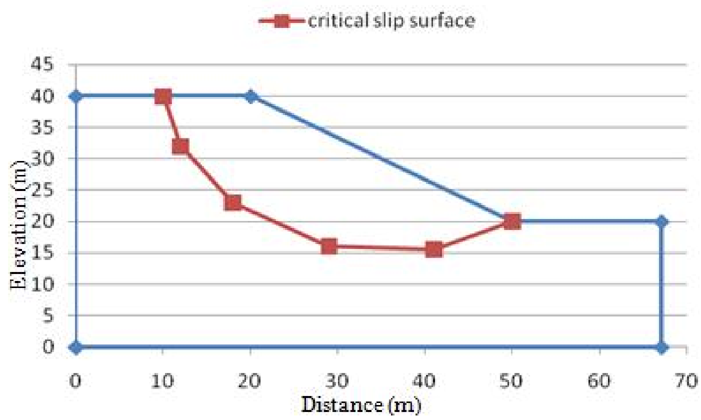

Figure 4. Critical slip surface in example 1

Table 1. Comparison the initial slip surface and critical line segment slip surfacee

	Critical circular slip surface	Critical line segment slip surface	Percent of improvement
Present method	1.322	1.15	13

Example 2: This example considers a slope of a homogeneous soil with pore water pressure (Figure 5) and seismic coefficient equal to 0.2.

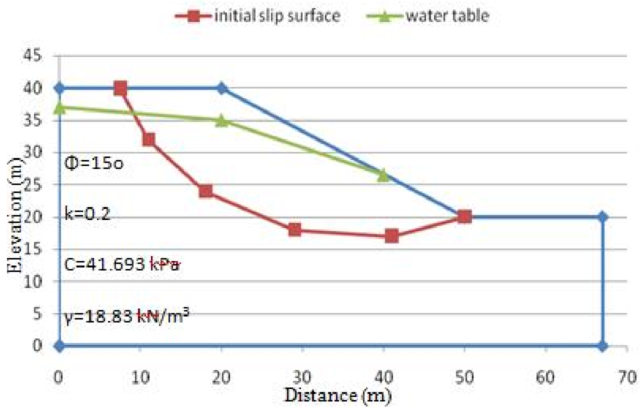

Figure 5. Properties and initial slip surface and water table in example 2

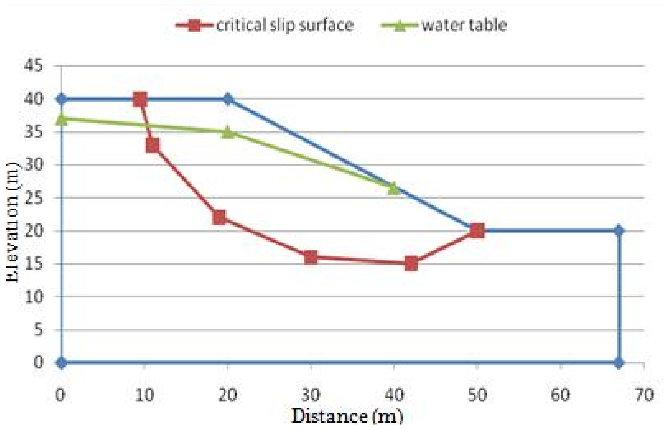

Figure 6. Critical slip surface in example 2

The values of the safety factor are determined from Equation 1 and critical slip surface is obtained by AVLG method. It first, is founded critical circular slip surface using search method (FS=1.088). Then 15 nodes are selected on the critical circular slip surface. It is an initial line segment slip surface to optimize using AVLG method. Then the initial slip surface is optimized after 6 trials (FS=0.98). Figure 6 shows the final circular and line segment critical slip surface, which gives the global minimum factor of safety. Table 2 demonstrates the results that are obtained by the present procedure. The minimum factor of safety for circular slip surface is 1.088 and minimum factor of safety, which is obtained by the present procedure, is 0.98 indicating that factor of safety is improved about 10 percent.

Table 2. Comparison of initial safety factor and critical line segment safety factor

	initial slip surface	critical line segment slip surface	Percent of improvement
Present method	1.088	0.98	10

CONCLUSIONS

This paper has developed the location of two-dimensional non-circular critical pseudostatic slip surface in earth slope using optimization method. The authors have extended the Alternating Variable Local Gradient (AVLG) method (that it is based on Univariate method) for global two-dimensional optimization analysis. This approach has successfully determined the location of critical slip surface. The results show the approach can be successfully employed to locate the critical slip surface in an earth slope. The AVLG procedure is very simple and it can be easily used. However it will not converge rapidly to the optimum solution and it has a tendency to oscillate with steadily decreasing progress toward the optimum. Hence it will be better to stop the computations at some point near to optimum point rather than trying to find the precise optimum point. This procedure can be described by the following steps:

- ✓ Find the critical circular slip surface using Grid search method or other methods as initial slip surface.
- ✓ Put the control nodes on initial slip surface, especially in the weak layers for stratified soils.
- ✓ Optimize the control nodes using Univariate method to obtain less factor of safety at the first cycle.
- ✓ Optimize the new control nodes using Univariate method to obtain less factor of safety at the next cycle.
- ✓ Repeat the step 4 while the difference between FS for two last slip surface to be lower than 1e-5.
- ✓ The latest factor of safety is minimum factor of safety and latest slip surface is critical slip surface.

Results indicate that AVLG method is sensitive to the number of control nodes and the initial slip surface. Numerical studies have demonstrated the efficiency of the present method for locating critical pseudostatic slip surface and corresponding minimum factor of safety in various conditions of slopes.

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