UPPER BOUND SEISMIC STABILITYANALYSIS OF EMBEDDED FOOTINGS

Seyyed Mojtaba Mousavi Orimi
M.Sc. Student of Geotechnical Engineering, Department of Civil Engineering, Qum University of Technology, Iran
Mousavi.Mojtaba@rocketmail.com

Morteza Jiryaei Sharahi
Assistant Professor of Geotechnical Engineering, Department of Civil Engineering, Qum University of Technology, Iran
jiriay@qut.ac.ir

Hossein Rezaeii
M.Sc. Student of Geotechnical Engineering, Department of Civil Engineering, Qum University of Technology, Iran
Rezaeii.Hossein@Gmail.com

Keywords: Pseudo-Static, Bearing Capacity, Strip Footings, Prandtl, Upper Bound

ABSTRACT

The paper pertains to the pseudo-static seismic stability analysis for finding the bearing capacity of embedded strip footings subjected to both horizontal and vertical earthquake forces using upper bound limit theorem assuming single-sided Prandtl-like failure mechanism. The least upper bound value is found using optimization technique. Results are obtained by developing a MATLAB computer code, which is validated by comparing the results with similar solutions available in literature. The comparison shows that the obtained results for surface footings agree well with the available solutions. Bearing capacity is observed to increase with embedment ratio; for a given embedment ratio, it decreases with increasing seismic acceleration coefficient. These are in tune with the expected results.

INTRODUCTION

Bearing capacity of foundations is one of the most interesting and widely studied topics in geotechnical engineering. Unreinforced footings are used commonly as foundations for different structures, provided it suffices the bearing capacity requirements. For unreinforced soils several studies have been conducted in the past for the estimation of bearing capacity of strip footings, beginning with Prandtl (1921), Terzaghi (1943), Meyerhof (1951) and so on. Different approaches such as, limit equilibrium method, limit analysis and method of characteristics etc. have been used for such analyses.

Limit analysis is being increasingly used for analysis of various stability problems in geotechnical engineering. Limit theorems proposed by Drucker, Prager and Lundgren (1953) facilitate to obtain the upper as well as lower bound values of the exact solution, which bracket the true collapse load. Here upper bound approach is made use of to evaluate the upper bound solution for bearing capacity of strip footings. Bearing capacity of shallow surface strip footings resting on unreinforced foundation beds has been extensively studied using this approach for static as well as seismic cases (Chen (1975), Michalowski (1997), Soubra (1999), etc). In this paper, upper bound limit analysis has been used to find the bearing capacity of footings, surface as well as shallow embedded strip footings, considering the seismic effects.

ANALYSIS

Bearing capacity embedded strip footing with width and depth of embedment as B_f and D_f respectively is considered. Application of larger loads on the footing may lead to bearing capacity failure of soil below. The bearing capacity failure is considered to be taking place along an assumed failure surface. In the present analysis, a single-sided Prandtl-like failure mechanism is assumed as shown in figure 1. The mechanism consists of three zones - a triangular active wedge, a log-spiral radial shear zone and a triangular passive wedge. The effect of earthquake forces has been considered using the pseudo-static approach of analysis.

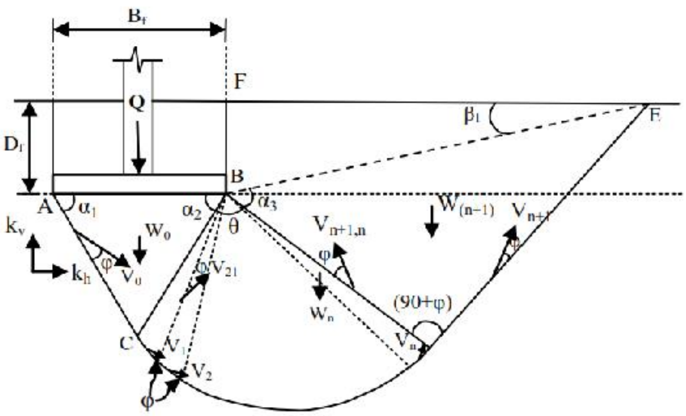


Figure 1. Failure mechanism

The following assumptions are made in analysing the problem:

- The soil is homogenous and isotropic
- Loading is uniform and vertical.
- Ground surface is horizontal.
- Footing is rigid and has rough base.
- Soil is an elastic-perfectly plastic material.
- Plane strain condition is satisfied.
- Pseudo-static analysis is valid.
- Shear strength of the soil is governed by Mohr-Coulomb failure criterion.
- Shear strength parameters are not affected by earthquake forces.

Bearing Let Q be the total ultimate load acting on the footing. The failure mechanism consists three zones - a triangular active zone beneath the footing (ABC), a logarithmic spiral radial shear zone (BCD) and a passive triangular wedge (BDE). Let ∠ BAC = α_1, ∠ ABC = α_2, ∠ DBE = α_3, ∠ FEB = β_1. Let the total central angle of the logarithmic zone, ∠ CBD=θ. The logarithmic zone, BCD is assumed to be divided into n triangular sectors, each having a central angle of θ/n. Let BC= r_0, BD= r_n. Then by the property of logarithmic spiral, $r_n = r_0 * e^{\theta \tan\varphi}$. DE and AC are assumed to be tangential to the spiral.

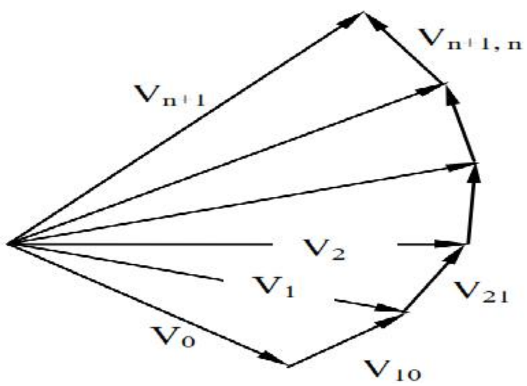


Figure 2. Velocity hodograph

Let V_0 be the velocity of wedge ABC, and V_1 to V_n are respectively the velocities of each triangular wedge inside BCD and V_{n+1} that of BDE. V_{10}, V_{21}, ..., $V_{n+1,n}$ are the relative velocities of $(i+1)^{th}$ space block with respect to i^{th} block (i=0,1,2,...,n) along the interface of two adjacent triangular blocks in log spiral zone.

All these velocities are inclined at an angle φ to the corresponding velocity discontinuities. The velocity hodograph is illustrated in figure 2. Pseudo - static approach is used to account for the seismic forces. In this approach, the earthquake forces are represented by equivalent static forces equal to the product of seismic coefficients and weight of each block.

Let k_h and k_v be the seismic acceleration coefficients in the horizontal and vertical directions respectively. Direction of k_h is taken from left to right towards the direction of soil block within the failure surface. Direction of k_v can be taken either upward or downward, in general. Here k_v is assumed in upward direction since that is the critical case.

These seismic accelerations produce inertia forces, k_hW and k_vW in the soil mass and its effect on external load is taken care by considering the net vertical load on footing as (1- k_v) Q and base shear k_hQ along the foundation base. Let W_0 is the weight of wedge ABC, W_i is the weight of each triangular block in log spiral zone and W_{n+1} is that of the wedge BDE and BEF. Inertia force acting due to the effect of seismic force in each block is k_hW horizontally and k_vW vertically, where W is the weight of the corresponding block. These inertia forces are assumed to act at the centroid of each block. Thus, the net force acting in each block is k_hW horizontally and (1- k_v) W vertically downwards.

Upper bound limit analysis is done by equating external work done and internal work done to find the ultimate load. The external work done (EWD) and the internal work done (IWD) are estimated as follows. External work done is computed as the product of force and velocity component in the direction of force. Internal work done is the energy dissipated internally, which is the product of cohesion, length of discontinuity and velocity component along the discontinuity, along all velocity discontinuities.

$$\begin{aligned} Total\ EWD = & \{(1-k_v)Q\cdot V_0\cdot\sin(\sim_1-\text{¤})+k_hQ\cdot V_0\cdot\cos(\sim_1-\text{¤})\}+ \\ & \{(1-k_v)W_0\cdot V_0\cdot\sin(\sim_1-\text{¤})+k_hW_0\cdot V_0\cdot\cos(\sim_1-\text{¤})\}+\left\{\sum_{i=1}^{n}\left[(1-k_v)W_iV_i^v+k_hW_iV_i^v\right]\right\}+ \\ & \{(1-k_v)W_{n+1}V_{n+1}^v+k_hW_{n+1}V_{n+1}^v+(1-k_v)q(EF)V_{n+1}^v+k_hq(EF)V_{n+1}^v\} \end{aligned} \tag{1}$$

$$Total\ IWD = c\times\left\{\sum_{i=0}^{n+1}\sim_i V_i\ \mathbf{cos}\text{¤}+\sum_{i=0}^{n}b_iV_{i+1,i}\ \mathbf{cos}\text{¤}\right\} \tag{2}$$

should Equating total external work done and total internal work done, we can find Q and hence the bearing capacity, q_u=Q/B. Among the various possible solutions obtained by using different values of the parameters, the least upper bound value is the best solution. Hence the objective function (bearing capacity) is optimized with respect to the geometrical parameters of the failure mechanism and the optimal value, which is the least possible bearing capacity, is obtained. The design vector (D) with respect to which the objective function is being optimized is:

$$D=\begin{bmatrix}\sim_1\\ \sim_2\\ \_\\ \\ \sim_3\end{bmatrix} \tag{3}$$

Equations Ultimate load, Q and hence the bearing capacity, q_u is a function of D i.e., q_u = f(D). Minimizing q_u = f(D), subjected to constraints of design parameters gives the upper bound solution. The optimization is done by using the „fmincon" function of the optimization toolbox of MATLAB 8.1, which helps in constrained non linear optimization of the objective function.Constraints used in optimization are as follows :

- α_2+ θ+ α_3 = 180, for D_f=0; α_2+ θ+ α_3 <270 and α_2+ θ+ α_3 >180, for $D_f \neq 0$, this is to ensure that mechanism is physically feasible.
- α_1+ α_2=(90+φ), based on properties of logspiral curve
- α_1+α_2<180, to satisfy the property of triangle, i.e., sum of interior angles in a triangle (ABC) is 180^o
- α_3 < (90- φ), to satisfy the property of triangle, i.e., sum of interior angles in a triangle (BDE) is 180^o
- $\alpha_1 > \varphi$, since if $\alpha_1 \leq \varphi$, V0 becomes horizontal or inclined above horizontal which is not possible.
- $5 \leq \alpha_1 \leq 85 - (\varphi/2)$; $5 \leq \alpha_2 \leq 85$; $5< \theta <180$; $5 \leq \alpha_3 \leq 85$. These are given based on the
- physical feasibility of failure mechanism geometry. If these constraints are not placed, during the optimization process the design parameters may have values, which make the failure mechanism physically untenable. The upper and lower bound on the variables has been placed based on experience gained while doing the computations.
- α_2+ θ+ α_3 -180 – sin-1(D_f/(BE)) =0. This is to make the β_1 angle consistent with both optimised angles and footing depth parameter.

Thus, a generalized formulation for the bearing capacity of embedded strip footings subjected to seismic forces has been devised using upper bound limit analysis and corresponding computer code is developed in MATLAB.

publications should be separated by a comma. Conference and journal titles should be written in italic, book, thesis and reports should be given underlined. Point should not be used within References.

RESULTS AND DISCUSSION

VALIDATION OF THE DEVELOPED METHOD

The results as obtained for unreinforced surface footing case, for both static and seismic conditions have been compared with the results available in literature so as to ensure the correctness of the developed approach. Convergence studies are conducted to determine the number of sectors required in log-spiral zone so that the bearing capacity values remain unaffected, Figures 3 shows that beyond 100 elements in the radial shear zone, the solution is not affected much. Therefore, all the results hereafter are obtained with 100 elements in the log spiral zone.

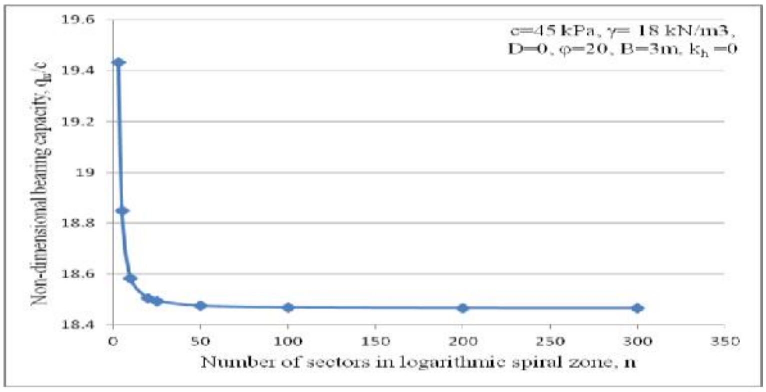


Figure 3. Schematic Convergence of solution (φ=20)

Equations Bearing capacity of unreinforced surface footings under static case is considered for comparison with the corresponding solutions obtained by Chen (1975). Comparison of dimensionless bearing capacity factors N_c, N_q, N_γ values obtained from present analysis for unreinforced surface footing with that obtained by Chen (1975) are shown in the tables 1, 2 and 3 and comparison of N_γ values with other various available results is shown in tables 4 and 5. From tables 1, 2 and 3 it is seen that the N_q values as predicted by Chen (1975) are very close to the present solution matching excellently well with each other where as the N_c Values differ from the present solution ranging from abour 0.2% to about 7.4% (when the soil friction angle is 40^o).

Table 1. Comparison of N_c values (unreinforced surface footing)

No.	Soil friction angle, φ	N_c (present values)	N_c (Chen, 1975)	Percentage error (%)
1	0	5.1496	5.14	-.01868
2	10	8.3177	8.347	0.3510
3	20	14.5736	14.84	1.7951
4	30	28.9019	30.15	4.1396
5	40	69.7991	75.34	7.3545

Table 2. Comparison of N_q values (unreinforced surface footing)

No.	Soil friction angle, φ	N_q (present values)	N_q (Chen, 1975)	Percentage error (%)
1	0	1	1	0.0000
2	10	1.5676	1.568	0.0255
3	20	2.4706	2.472	0.0566
4	30	6.3893	6.401	0.1828
5	40	18.3272	18.41	0.4498

Table 3. Comparison of N_γ values (unreinforced surface footing)

No.	Soil friction angle, φ	N_γ (present values)	N_γ (Chen, 1975)	Percentage error (%)
1	0	0	0	0.0000
2	5	0.2643	0.382	30.8115
3	10	0.921	1.16	20.6034
4	15	2.3234	2.73	14.8938
5	20	5.2309	5.87	10.8876
6	25	11.3696	12.4	8.3097
7	30	24.9141	26.7	6.6888
8	35	56.871	60.2	5.5299
9	40	139.5905	147	5.0405

However, it is a bit surprising that the N_γ values obtained from the present solution differ by large amounts for lower values of soil friction angle in comparison to higher values of soil friction angles. But on the whole the absolute values are close enough to consider the solutions to be acceptable with the possible margin of errors due to the assumptions involved and the assumed mechanism of failure.

Table 4. Comparison of N_γ values with different results in literature (limit analysis)

No.	Soil friction angle, φ	N_γ (present values) UB	Ukritchon et.al. (2003) LB	Ukritchon et.al. (2003) UB	Michalowski (1997) UB	Soubra (1999) UB	Hjiaj (2005) LB	Khatri (2008) LB	Khatri (2008) UB
1	5	0.2643	0.11	0.12	0.181	-	0.12	0.11	0.13
2	10	0.921	0.41	0.47	0.706	-	0.43	0.4	0.48
3	15	2.3234	1.13	1.31	1.938	1.95	1.18	1.09	1.32
4	20	5.2309	2.67	3.27	4.468	4.49	2.82	2.65	3.16
5	25	11.3696	5.95	7.52	9.765	9.81	6.43	6.02	7.26
6	30	24.9141	13.2	17.4	21.394	21.51	14.57	13.65	16.54
7	35	56.871	29.3	42.4	48.681	49	33.95	31.9	38.96
8	40	139.5905	69.9	111.1	118.827	119.84	83.33	77.88	98.53

Table 5. Comparison of N_γ values with different results in literature

No.	Soil friction angle, φ	N_γ (present values) UB	Bolton and Lau (1993) stress characteristics	Meyerhof (1963) Semi empirical	Terzaghi (1943) Limit equilibrium	Vesic (1973) Limit equilibrium	Booker (1969) stress characteristics
1	5	0.2643	0.62	-	0.1	0.45	-
2	10	0.921	1.71	-	0.7	1.22	0.46
3	15	2.3234	3.17	-	2	2.65	1.15
4	20	5.2309	5.97	2.87	4.8	5.39	2.89
5	25	11.3696	11.6	6.77	9.8	10.9	6.63
6	30	24.9141	23.6	15.7	20	22.4	15
7	35	56.871	51	37.2	43	48	35.8
8	40	139.5905	121	93.7	-	109	83.8

All Comparison of the results presented in Tables 4 and 5 shows that the solutions differs from each other greatly and it is difficult to draw any definite conclusions regarding which one is more correct. But the results presented in Table 5 show that the present solutions are closer to those of Bolton and Lau (1993) and Vesic (1973) obtained by using method of stress characteristics and limit equilibrium.

Figure 4 show comparisons of present results obtained for seismic case with those available in literature and it can be seen that the present solutions have a reasonably good agreement with the available results. Thus it is seen that the results obtained from the present analysis are in good agreement with the known solutions for unreinforced surface footing case ensuring the validity of the developed computer code establishing it to be correct.

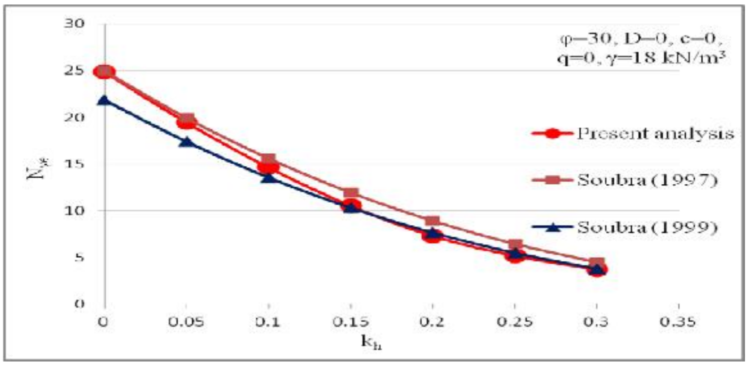


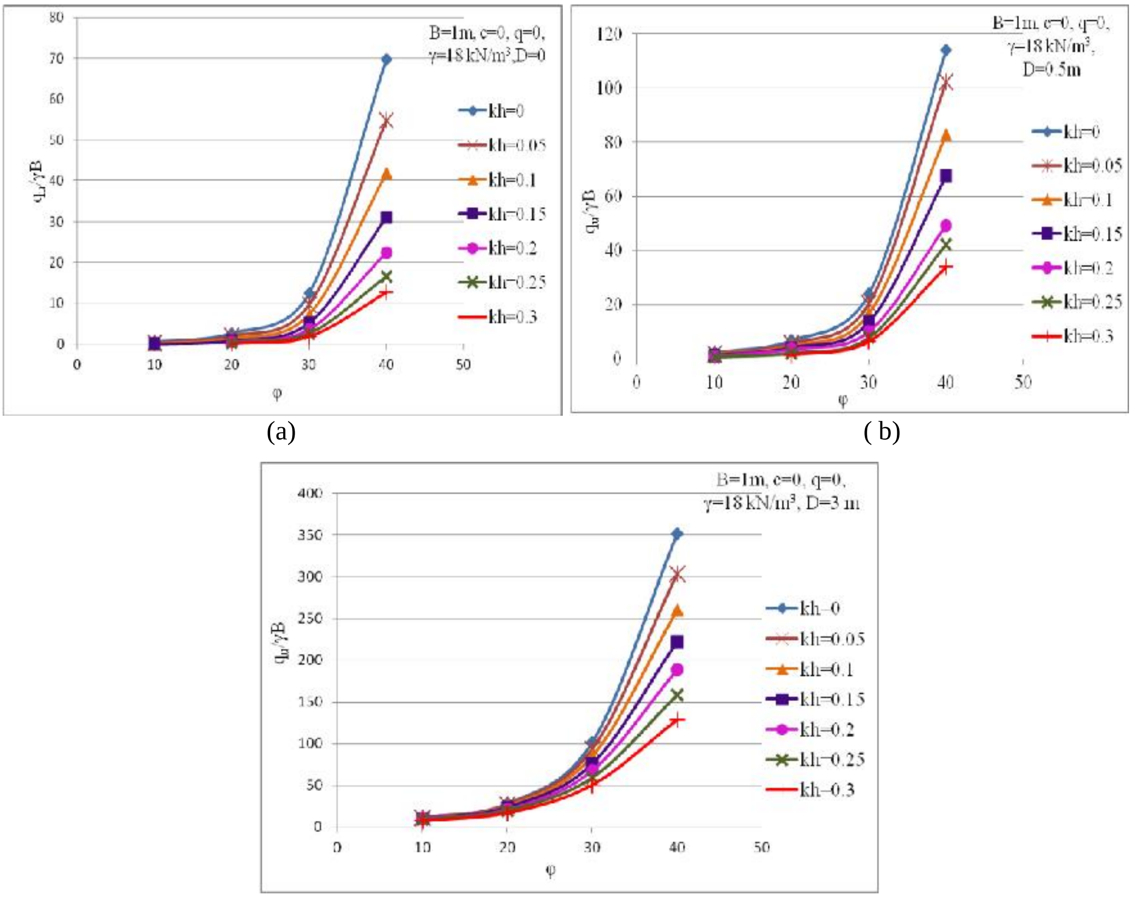
Figure 4. Comparison of seismic bearing capacity factor $N_{\gamma e}$ with available solutions

EFFECT OF EMBEDMENT

The increase in bearing capacity due to embedment and the effect of seismic forces on the bearing capacity of unreinforced embedded footing is studied. Figures 5(a,b,c) show the variation of non-dimensional bearing capacity, $q_u/\gamma B_f$ with soil friction angle, φ for different values of horizontal seismic coefficient, k_h and for different embedment ratios.

It can be seen from figure 5(a) that for surface footings, the non-dimensional bearing capacity ($q_u/\gamma B_f$) increases with the increase in the value of the friction angle φ. For a given φ as k_h increases the bearing capacity decreases. The rate of decrease is more pronounced for higher φ-values. Similar trend of behaviour is also observed for embedded footings with D_f/B_f ratio varying from 0.5 to 3, which is shown in figures 5(a,b).

Figure 6 highlights the effect of the embedment ratio on $q_u/\gamma B_f$. From the figure, it is seen that with increase in D_f/B_f ratio, $q_u/\gamma B_f$ increases for the same k_h and φ value, and with increase in k_h, bearing capacity decreases for the same embedment ratio.



(a) (b)

(c)

Figure 5. Variation of bearing capacity with soil friction angle, φ for various k_h (D_f/B_f: a=0 , b=0.5 , c=3)

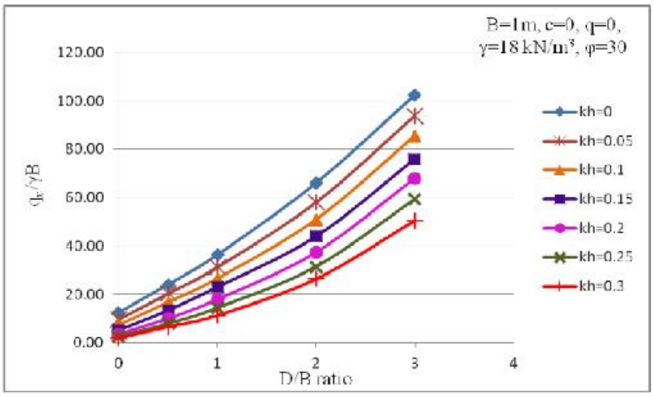


Figure 6. Variation of bearing capacity with embedment ratio for various k_h (φ=30)

CONCLUSIONS

An analysis method for bearing capacity of embedded strip footings has been devised using upper bound approach of limit analysis; computer code is developed for the same and validated with available results in literature. Validation of results is done for the unreinforced surface footing case. The results obtained for the static as well as seismic case of unreinforced surface footing agrees well with the available results.

Unreinforced bearing capacity is observed to increase with embedment ratio; for a given embedment ratio, it decreases with increasing seismic acceleration coefficient. Both the above observations are in tune with the general expectation. The quantitative values can be read from the graphs for use in designs.

REFERENCES

ASCE, 99(1), (1973), 45–73.

Bolton MD and Lau CK (1993) Vertical bearing capacity factors for circular and strip footings on Mohr–Coulomb soil, *Can. Geotech. J., 30*, 1024–1033

Booker JR, Manoharan N and Dasgupta SP (1969), "Bearing capacity of surface footings by finite elements", *Computers and Structures*, Vol. 54, No. 4, pp. 563-586

Chen WF (1975) " Limit analysis and soil plasticity", Elsevier, Amsterdam

Choudhury D and Subba Rao KS (2005) "Seismic Bearing Capacity of Shallow Strip Footings", *Geotechnical and Geological Engineering*, Vol. 22, pp. 403-418

Farzaneh O, Ganjian N and Askari F (2010), "Rotation-translation mechanisms for upper-bound solution of bearing capacity problems", *Computers and Geotechnics*, Vol. 37, pp. 449-455

Gourvenec S (2008), "Effect of embedment on the undrained capacity of shallow foundations under general loading", *Geotechnique*, Vol. 58, No.3, pp.177-185

Hjiaj M, Lyamin AV and Sloan SW (2005) "Numerical limit analysis solutions for the bearing capacity factor Nγ", *International Journal of Solids and Structures*, Vol. 42, pp. 1681-1704

Khatri VN, Kumar J and Kouzer KM (2008) "Determination of the bearing capacity factor Nγ using upper and lower bound limit analysis including slip line method", *The 12th International Conference of International Association for Computer Methods and Advances in Geomechanics*

Meyerhof GG (1951) "The ultimate bearing capacity of foundations", Geotechnique, Vol. 2, pp. 301-332

Michalowski RL (1997), "An estimate of the influence of soil weight on bearing capacity using limit analysis", *Soils and Foundations*, Vol. 37, No. 4, pp. 57-64

Soubra AH (1999) "Upper bound solutions for bearing capacity of foundations", *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, Vol. 125, No. 1, pp. 59-68

Terzaghi K (1943) "Theoretical SoilMechanics", Wiley, Newyork

Ukritchon B, Whittle AW and Klangvijit C (2003) "Calculations of bearing capacity factor Nγ using numerical limit analyses", *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 129, No. 6, pp. 468–474

Vesic AS : Analysis of ultimate loads of shallow foundations, *J. Soil Mech. and Found. Div*

Wang, YJ, Yin JH and Chen ZY (2001) "Calculation of bearing capacity of a strip footing using an upper bound method", *International Journal for Numerical and Analytical Methods in Geomechanics*, Vol. 25, pp. 841-851