

## RELIABILITY BASED TOPOLOGY OPTIMIZATION FOR THE SEISMIC DESIGN OF TRUSS-LIKE STRUCTURES

Hasan MOGHADDAM

*Professor, Sharif university of Technology  
moghadam@sharif.edu*

Afshin MOHAMMADI

*Sharif university of Technology, International campus  
afsh\_mohammadi@yahoo.com*

**Keywords:** Topology Optimization, Seismic Design, Truss-Like Structures

### ABSTRACT

In this paper, reliability based design methodology is applied towards optimization of truss structures excited by earthquake ground motions. The total weight of structure is optimized under constraints related to minimum target reliabilities specified for each element and different performance requirements. To achieve this goal, the redundant materials slightly move from strong spots to the weak segments of the structure until a state of uniform deformation and confidence prevails. Probabilistic seismic performance assessment adopted by PEER is applied to calculate the expected mean annual exceedance frequency of demand parameter of a given truss system considering seven seismic excitations. The efficiency of the applied method is illustrated by case study. The algorithm has the capability of considering desired reliability constraints for each element resulting in a balanced distribution of weight.

### INTRODUCTION

Direct guidance for reducing the potential for damage to the structural systems during a building's service life cannot be provided by conventional building codes (Rojas, Foley, & Pezeshk, 2011). Some procedures to overcome this restriction in the context of performance-based seismic design are posed, ((FEMA356, 2000);(ATC40, 1996)). In addition, Pacific Earthquake Engineering Center (PEER) puts forward a performance-based design procedure in a reliability format evolved for evaluation of structures. Four sources of uncertainties due to seismic hazard, structural demand, structural capacity, and seismic induced consequences are combined aiming to estimate mean annual frequency of decision variables in terms of direct financial loss and number of casualties. To achieve this very end, four steps entitled as seismic hazard analysis, structural analysis, damage analysis, and loss analysis are considered. The PEER approach which is formulated based on the application of the total probability theorem, is presented in equation (1):

$$\lambda(DV) = \iiint G(DV | DM).dG(DM | EDP).dG(EDP | IM).d\lambda(IM) \quad (1)$$

In the formula above,  $\lambda(DV)$  represents the probabilistic description of the decision variables, DM shows the damage measure, EDP represents the engineering demand parameter, and IM is the intensity measure. The main goal of the PEER methodology is to merge all significant sources of uncertainty that appear up in specification of the ground motion, the material properties, and the modeling and evaluation process.

Achieving optimized topology of structures through reliability-based design intends searching the best balance between cost reduction and safety assurance while various sources of uncertainties are controlled which cannot be achieved by deterministic optimization (Chateaufneuf, 2008)

Various studies are conducted towards performance-based optimization of structures considering deterministic constraints ((Ganzerli, Pantelides, & Reaveley, 2000); (Liu, Burns, & Wen, 2003); (Liu, Burns, & Wen, 2005); (Zou, Chan, Li, & Wang, 2007); (Hajirasouliha, Moghaddam, & Pilakoutas, 2011)). Furthermore, some researches are extended to consider probabilistic constraints ((Rojas et al., 2011); (Fragiadakis & Papadrakakis, 2008); (Foley, Pezeshk, & Alimoradi, 2007); (Lagaros, Garavelas, & Papadrakakis, 2008)). There is an appropriate time to present the topology optimization methods in view of probabilistic reliability-based manner.

This paper addresses the recent results of PEER framework in topology optimization of truss structures as a part of mentioned researches. A numerical model of sampled truss structure is optimized considering a level of mean annual frequency of exceedance as target reliability. The final structure is to have a minimum weight while achieving topology and size optimized solution as well as the desired reliability. Nonlinear time history analyses are conducted to evaluate structural response excited by sets of natural record and target mean annual exceedance frequency corresponding to specified demand ductility are achieved.

## METHODOLOGY

A deep and scrutinized consideration of the different variables that appear in PEER framework illuminates that to integrate across three separate variables is not generally requisite. As an instance, a demand measure may be interpreted directly into a decision variable inasmuch as damage parameters and decision variables are adjacently connected to each other ((Kunnath, 2006)). With regard to the previous explanation, the EDP hazard curve will be established by considering the individual probabilities of demand in equation (2):

$$\lambda(EDP > edp) = \int P(EDP > edp | IM) . d\lambda(IM) \quad (2)$$

Mean annual frequency of exceedance (MAFE) which is evaluated based on equation (2), is affected by many uncertain parameters which their precise values are unknown. Therefore it is desirable to consider various sources of uncertainties towards evaluation of MAFE. It has been shown that uncertainty sources, other than variability of earthquake induced ground motions, have a negligible effect on the variability of structural response and MAFE ((Lee & Mosalam, 2005)). Therefore in this study the uncertainty inherent in induced ground motions are considered through nonlinear time history analysis of the structure under a set of natural recorded ground motions.

The iterative optimization procedure developed (Hajirasouliha, Pilakoutas, & Moghaddam, 2011) for topology optimization design of truss- like structures is extended for reliability optimization of non-linear truss structures. This method implements the PEER framework to acquire minimum weight under probability constraint. The weight objective function  $f$  to be minimized can be formulated as:

$$\text{Minimize: } f = \sum \rho_i l_i A_i \quad (3)$$

In the aforementioned equation, the design variables are the cross-sectional areas of truss members (signified as  $A$ ) and  $\rho_i$  and  $l_i$  are material density and length of  $i$ th member, respectively. In the nonlinear range of response with uncertainty, probability of member ductility ratio is an appropriate criterion for the aim of appraising seismic performance of truss structures subjected to seismic excitation. Hence, here the design variables are selected to meet design constraints as follows:

$$\lambda_i(A_1, A_2, \dots, A_n) \geq \lambda_{\min}^{el} \quad (4)$$

$\lambda_i$  connotes MAFE of  $i$ th member ductility and  $\lambda_{\min}^{el}$  is target MAFE of  $i$ th member ductility, respectively. Positions where the MAFE is larger than the target values are recognized and the inefficient material is reduced until an optimum state is obtained. To gain this goal, the following equation is applied:



$$[A_i]_{m+1} = [A_i]_m [SR_i]^\alpha \quad (5)$$

$$SR_i = \frac{\lambda_i}{\lambda_t} \quad (6)$$

Where  $[A_i]_m$  represents the cross-sectional areas of the  $i$ th member at  $m$ th iteration,  $\lambda_i$  and  $\lambda_t$  are maximum MAFE of specific ductility for the  $i$ th member and target MAFE, respectively,  $\alpha$  is the convergence parameter which suggested by (Hajirasouliha et al., 2011) to range from 0 to 1.

Relation of MAFE of spectral amplitude versus the spectral value is represented by the following analytical form in the hazard range of interest.

$$\lambda(S_a) = k_0 S_a^K \quad (7)$$

In which  $k_0$  and  $K$  are constants evaluated by regression analysis to seismic hazard data and  $S_a$  is spectral acceleration at fundamental period of the structure.

In the proposed method, the ductility ratios ( $\mu$ ) are utilized to evaluate element performance. The distribution of the ductility of element as a function of the intensity measure may be presented by functional form shown by equation (8).

$$e^{\eta_{\ln[\mu|S_a]}} = a(S_a)^b \quad (8)$$

Where  $\eta_{\ln[\mu|S_a]}$  is the mean of maximum ductility values (seismic demand parameter), of the member and  $a$  and  $b$  are regression constants.

It has been shown that evaluation of MAFE of demand parameter based on equation (2) can be estimated by equation (9).

$$\lambda(x) = k_0 \left[ \left( \frac{x}{a} \right)^b \right]^{-k} \exp \left[ 0.5k^2 \left( \frac{\sigma_{\ln(x|S_a)}}{b} \right)^2 \right] \quad (9)$$

Figure (1) shows a flowchart for topology optimization implemented in the current research effort within the context of proposed algorithm.

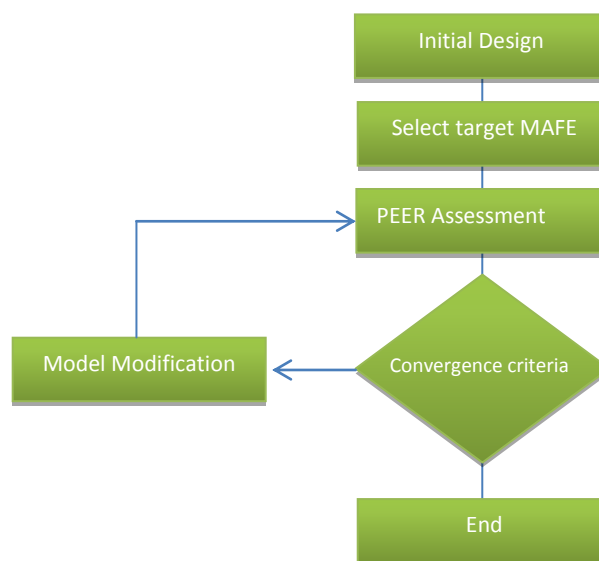


Figure 1. Topology optimization flowchart

## EXAMPLE

The usefulness and practicality of the optimization method, which is proposed in this paper, is elucidated by means of a conceptual example depicted in Figure 2. The primary object is to design a truss structure for holding four masses M1–M2–M3–M4 (supposed to be 20, 5, 5 and 10 tons, respectively) by utilizing any number of members bonding these masses to each other and to the supports. Inertia effects, which are induced seismically, are the mere forces that act on points. It is presumed that the yield strength of each member is equal in tension and compression. Nonlinear dynamic analyses were executed using OpenSees. The assumed horizontal ground motion acceleration records used are shown in Table (1).

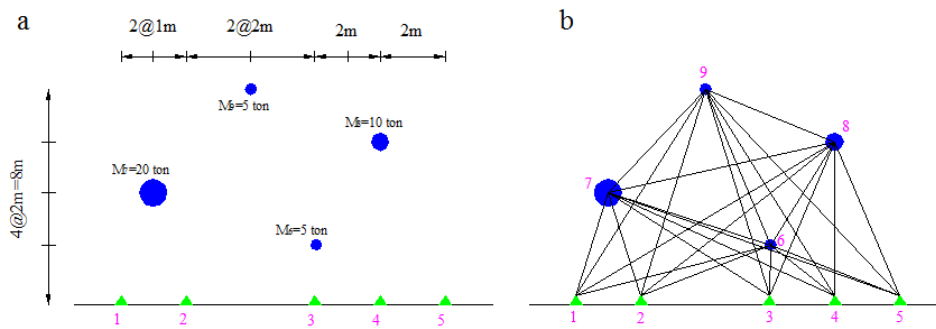


Figure 2.(a) Position of masses and supports. (b) Joint numbers and preliminary arrangement of members

Table 1. Ground motion acceleration records

Event	Year	Station	M	Mechanism	Rjb(km)
Imperial Valley-06	1979	Superstition Mtn Camera	6.53	Strike-Slip	24
Morgan Hill	1984	Corralitos	6.19	Strike-Slip	23.2
Morgan Hill	1984	San Juan Bautista, 24 Polk St	6.19	Strike-Slip	27.1
Chi-Chi, Taiwan-04	1999	CHY029	6.2	Strike-Slip	25.8
Chi-Chi, Taiwan-04	1999	CHY034	6.2	Strike-Slip	28.4
Chi-Chi, Taiwan-04	1999	CHY035	6.2	Strike-Slip	25
Chi-Chi, Taiwan-04	1999	TCU084	6.2	Strike-Slip	26.8
Chi-Chi, Taiwan-04	1999	TCU089	6.2	Strike-Slip	27.2
Chi-Chi, Taiwan-04	1999	TCU116	6.2	Strike-Slip	28.7
Chi-Chi, Taiwan-04	1999	TCU122	6.2	Strike-Slip	23.1

## RESULT

Table 2 represents the cross-sectional area of truss members for two assumed target MAFEs (i.e.  $\lambda=0.0021$ ) corresponding to demand ductility of 2. The total weight of the truss for each step of optimization is shown in Figure (3).

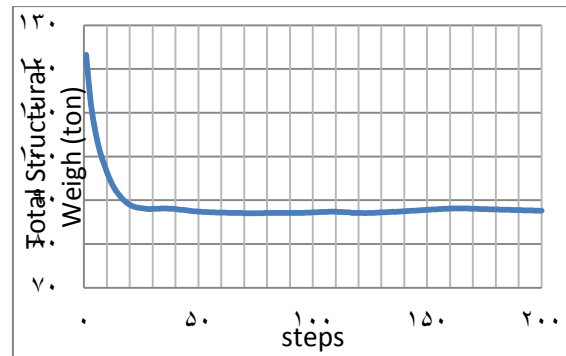
Figure 3. Total Weight ( $\lambda=0.0021$ )

Table 2. Members connectivity and optimum answer

Member	First Joint	Second joint	A ( $\lambda=0.0021$ ) cm <sup>2</sup>
1	1	6	0
2	2	6	1035.755
3	3	6	0
4	4	6	1168.626
5	5	6	0
6	7	6	1390.005
7	8	6	0
8	9	6	0
9	1	7	3172.684
10	2	7	0
11	3	7	1957.074
12	4	7	969.5594
13	5	7	0
14	8	7	213.2666
15	9	7	957.261
16	1	8	1209.221
17	2	8	1465.135
18	3	8	0
19	4	8	0
20	5	8	2178.084
21	9	8	578.5052
22	1	9	0
23	2	9	0
24	3	9	0
25	4	9	0
26	5	9	696.0876

The final weight of optimum structures for target MAFEs corresponding to demand ductility of 2 are given in Table 3.

Table 3. Optimum design for different targets of mean annual exceedance frequency of ductility demands

No.	1
$\lambda(\mu > 2)$	0.0021
optimal shape	
Final weight (ton)	180.92

For brevity variation of MAFE, cross sectional area and engineering demand parameter hazard curve of member No.9 towards the final solution are illustrated in Figures (4) and (6).

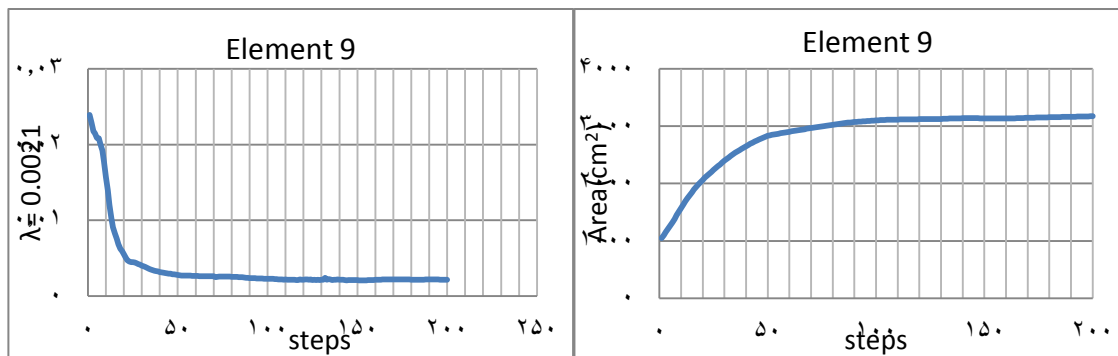


Figure 4. (a) Convergence to target MAFE (Element No.9). (b) Convergence to optimum area (Element No.9)

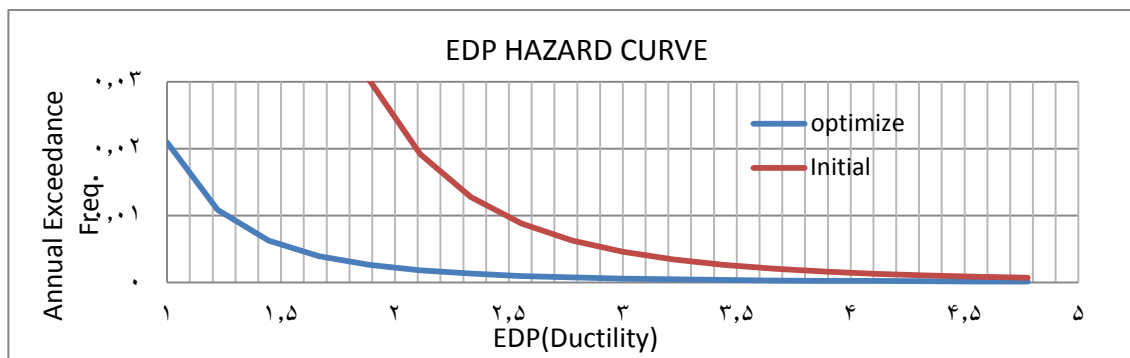


Figure 5. engineering demand parameter hazard curve of member No.9

## CONCLUSIONS

In this study, a practical optimization method is presented for reliability seismic design of truss structures. Based on the results, the concept of uniform confidence level of demand can be used efficiently for reliability-based optimization of nonlinear truss structures subjected to seismic excitations. The proposed algorithm may be considered as an effective tool for structural engineering profession towards design of structures based on presumed confidence level. This algorithm also allows easily comparing and contrasting the structural designs with conventional deterministic methodologies.

The results show that optimal topologies as well as calculated cross sectional areas are influenced by assumed target MAFE and therefore, a fixed arrangement of truss members cannot be appropriate for different confidence levels.

## REFERENCES

- ATC 40 (1996) Seismic evaluation and retrofit of existing concrete buildings. Redwood City (CA): Applied Technology Council
- Chateaufneuf A (2008) Principles of reliability-based design optimization. In Yiannis Tsompanakis, N. D. Lagaros & M. Papadrakakis (Eds.), Structural design optimization considering uncertainties (Vol. vol.1). London, UK: Taylor & Francis
- FEMA 356 (2000) Prestandard Commentary for the Seismic Rehabilitation of Buildings. Washington D.C.: Federal Emergency Management Agency.



- Foley C, Pezeshk S and Alimoradi A (2007) Probabilistic performance-based optimal design of steel moment-resisting frames. I: Formulation. *Journal of Structural Engineering*, 133(6), 757-766
- Fragiadakis M and Papadrakakis M (2008) Performance- based optimum seismic design of reinforced concrete structures. *Earthquake Engineering & Structural Dynamics*, 37, 825-844
- Ganzerli S, Pantelides CP and Reaveley LD (2000) Performance-based design using structural optimization. *Earthquake Engineering & Structural Dynamics*, 29(11), 1677-1690
- Hajirasouliha I, Moghaddam H and Pilakoutas K (2011) Topology optimization for the seismic design of truss-like structures. *Computers and Structures*, 89(7), 702-711
- Kunnath SK (2006) Performance-Based Seismic Design and Evaluation of Building Structures. In W. Chen & E. Lui (Eds.), *Earthquake engineering for structural design*
- Lagaros ND, Garavelas AT and Papadrakakis M (2008) Innovative seismic design optimization with reliability constraints. *Computer Methods in Applied Mechanics and Engineering*, 198(1), 28-41
- Lee T.-H and Mosalam KM (2005) Seismic demand sensitivity of reinforced concrete shear-wall building using FOSM method. *Earthquake Engineering & Structural Dynamics*, 34(14), 1719-1736
- Liu M, Burns SA and Wen YK (2003) Optimal seismic design of steel frame buildings based on life cycle cost considerations. *Earthquake Engineering & Structural Dynamics*, 32(9), 1313-1332
- Liu M, Burns SA and Wen YK (2005) Multiobjective optimization for performance-based seismic design of steel moment frame structures. *Earthquake Engineering & Structural Dynamics*, 34(3), 289-306
- Rojas HA, Foley C and Pezeshk S (2011) Risk-Based Seismic Design for Optimal Structural and Nonstructural System Performance. *Earthquake Spectra*, 27(3), 857-880
- Zou XK, Chan CM, Li G and Wang Q (2007) Multiobjective Optimization for Performance-Based Design of Reinforced Concrete Frames. *Journal of Structural Engineering*, 5(4), 234-256

