PERFORMANCE-BASED OPTIMUM DESIGN OF REINFORCED CONCRETE MOMENT FRAMES UNDER EARTHQUAKE LOADING

Mohsen TEHRANIZADEH
Professor, Amirkabir University of Technology, Tehran, Iran,
m.teh.2008@gmail.com

Fatemeh Sadat AKHAVAN HEJAZI
PhD. Student, Amirkabir University of Technology, Tehran, Iran,
fatahejazi@gmail.com

Keywords: Performance-Based Design, Design Optimization, Pushover Analysis, Concrete Moment Frame, Least Structural Weight, Uniform Ductility Demands

ABSTRACT

This paper presents an effective computer-based pushover analysis technique for the performance-based design of concrete frames to predict post-elastic seismic demands under equivalent static earthquake loading. Using an energy approach, the performance-based optimization of concrete moment resisting frames is evaluated for the so-called operational, immediate occupancy, life safety and collapse prevention performance levels. Three objective criteria are identified for the performance-based seismic design, which include the least structural weight, uniform ductility demands and also uniform earthquake energy for all the stories. The results obtained for three- and five-story concrete moment frames and compared with the dynamic behavior of these buildings.

INTRODUCTION

The concept of the performance-based structural design based on seismic loading conditions was introduced recently in the literature as (FEMA-273, 1997). To assess the structural performance, the guidelines recommend the use of various methods of analysis including linear static, nonlinear static, linear dynamic and nonlinear dynamic. It is common to use the pushover analysis method for its simplicity and its ability in estimating, with acceptable accuracy, the component, and system deformation demands, without the intensive computational and modeling efforts involved in a dynamic analysis. Therefore, in the performance-based structural design, a nonlinear static procedure is implemented in the analysis to estimate the seismic structural deformations.

In the performance-based design, the main objective is to consider the structural performance in resisting the earthquake loading in a quantifiable manner at various levels, and to achieve more predictable and reliable levels of safety and operability during natural hazards.

A design performance level expresses the desired behavior of the structure under the design earthquake loads. In the performance-based seismic design codes, different performance levels have been defined. The performance levels are categorized according to (FEMA-356, 2000) as: operational (OP), immediate occupancy (IO), life safety (LS) and collapse prevention (CP).

Perhaps Galileo was the first scientist who proposed the structural optimization idea in 1638 through the uniform strength criterion for a bent beam. This work was followed by other researchers such as (Maxwell, 1980).

Performance-based optimum design of reinforced concrete buildings is a relatively new field of research. The performance criteria which are imposed as constraints, affect the initial construction cost that has to be minimized. Based on this approach, perhaps the first effort to combine the contemporary concept of performance-based design with structural optimization came from (Ganzerli, 2000). They proposed an optimization methodology for seismic design, considering the performance-based constraints. Lagaros (2006) proposed an automated procedure for minimizing the eccentricity between the mass center and the rigidity center in RC structures. Furthermore, in the work by (Li and Cheng, 2001), the optimal decision model of the target value of performance-based structural system reliability of RC frames is established according to the cost-effectiveness criterion.

In this paper, the performance-based seismic design of RC buildings using an optimization procedure is automated. To this end, the objective functions which have been used by Xu (1994) are reviewed with some modifications in the lateral load distribution, and also another objective function for unifying the story energy is presented. Optimization MATLAB® toolbox is used to find minimum of constrained nonlinear multivariable function. Finally, the FEDEASLab toolbox (2004) is utilized to study the responses of the optimized multistory structures, in a nonlinear dynamic analysis with elements of distributed inelasticity.

NONLINEAR STATIC ANALYSIS (PUSHOVER)

The Pushover analysis is a simple procedure for evaluating the response of a structure to the incremental lateral loadings. This procedure monitors the progressive stiffness degradation of a frame as it is loaded into the post-elastic range of behaviour. In this type of analysis, the lateral load distribution along the height of the frame is very important. For this reason, FEMA-273 (1997) has defined the lateral load distribution as follows:

$$P_s = C_{v,s} V_b = \frac{G_s H_s^{\mu}}{\sum_{k=1}^{ns} G_k H_k^{\mu}} V_b, \tag{1}$$

Where Ps is the lateral load at the story level s; Cv,s is the lateral inertial load distribution factor; Vb is the base shear; Hs and Hk are the heights from the base of the building to the story levels s and k, respectively; Gs and Gk are the seismic weights for the story levels s and k, respectively; and μ is a constant number determined by the fundamental period of the frame calculated in each optimization step according to FEMA-273 (1997):

$$\mu = \begin{cases} 2 & T \geq 2.5 \\ 0.5T + 0.75 & 0.5 < T < 2.5 \\ 1 & T \leq 0.5 \end{cases}, \tag{2}$$

Here T is the fundamental elastic period of the structure. In the nonlinear static analysis, the distribution of lateral loads along the height of frame is determined by the fundamental elastic period of the structure. Since the elastic period is kept constant in each step of the optimization analysis and varies from one step to another, the load distribution should be modified during the analysis. Therefore, the structure's elastic period is determined in each step of the optimization and consequently the load distribution is updated.

To predict the seismic demands of a building's frameworks under equivalent static earthquake loading, Xu (1995) proposed a new computer-based pushover analysis procedure. This procedure was originally conceived for the elastic analysis of steel frameworks with semi-rigid connections. Monfortoon and Wu (1963) modelled the connection at each end of a member as a linear spring and introduced the non-dimensional 'rigidity-factor', r, as:

$$r = \frac{1}{1 + \frac{3EI}{RL}}, \tag{3}$$

Where R is the rotational stiffness of the connection, and EI and L are the bending stiffness and the length of the connected member, respectively. The rigidity-factor is defined as the connection's stiffness relative to that of the attached member.

The influence of the semi-rigid connection on the overall performance of a frame structure under increasing loads is accounted for through an incremental load analysis, presuming the nonlinear moment-rotation relations which characterize the variation in the rotational stiffness of the semi-rigid connections under increasing moment (Xu, 1995). During the calculation of each finite load increment, the rotational stiffness, R, of each connection is assumed to be constant, and the rigidity-factor, r, is obtained through equation (3). Member stiffness matrices are obtained through equation (4) and provide the corresponding increments of the element moments and rotations:

$$K_j = S_j.C_{sj} + G_j.C_{gj}\,, \tag{4}$$

Here Gj and Cgj are the standard geometric stiffness matrix and the corresponding correction matrix, respectively; Sj is a standard elastic stiffness matrix for the member having 'rigid' end-connections; and Csj is a correction matrix (see the appendix).

To monitor the progressive plastification (stiffness degradation) of the frame members under increasing loads, a plasticity factor can be used. To this end, a nonlinear moment-rotation (M-φ) relation that characterizes the variation in post-elastic flexural stiffness of a plastic-hinge section under increasing moment, figure (1), should be adopted.

$$M(\varphi) = M_y + \sqrt{(M_P - M_y)^2 - [(M_P - M_y)(\varphi_P - \varphi)/\varphi_P]^2}\,, \qquad \varphi \le \varphi_P \tag{5}$$

Upon differentiating equation (5) with respect to φ, the post-elastic flexural stiffness, R^P, is defined as:

$$R^P = \frac{dM(\varphi)}{d\varphi} = \frac{(M_P - M_y)^2(\varphi_p - \varphi)}{\varphi_P^2\sqrt{(M_P - M_y)^2 - [(M_P - M_y)(\varphi_P - \varphi_y)/\varphi_P]^2}}\,, \qquad \varphi \le \varphi_P \tag{6}$$

Where M_y, M_P are the first-yield and fully-plastic moment capacities of the member, and φ_P and φ are the fully plastic rotation and the extent of post-elastic rotation, respectively. According to the design variables, the values of M_y, M_P should be changed. As shown in figure (2), M_y for a moment hinge (no axial force) can be expressed in terms of the reinforcement ratios (Zou, 2005) as:

$$M_y = \frac{1}{2} f_c Bkd(\frac{kd}{3} - d') + f_y Bd(d - d')\rho\,, \tag{7}$$

Where f_c is the stress at the concrete's extreme compression fiber; f_y is the yield strength of the steel; ρ is the tension steel reinforcement ratio; B is the width of the concrete section; d is the effective depth; d' is the distance from the extreme compression fiber to the centroid of the compression steel; and k is the neutral axis depth factor at the first yield as expressed below:

$$k = \sqrt{(\rho+\rho')^2 n_{sc}^2 + 2(\rho + \rho'\frac{d'}{d})n_{sc}} - (\rho+\rho')n_{sc}, \quad \rho = \frac{A_s}{Bd}, \quad \rho' = \frac{A_s'}{Bd} \tag{8}$$

Where nsc is the ratio of the modulus of elasticity of steel to that of concrete, and ρ' is the compression steel reinforcement ratio. For simplicity, Mp can be approximately related to M_y (Zou, 2005) as:

$$M_p = 1.1 M_y \tag{9}$$

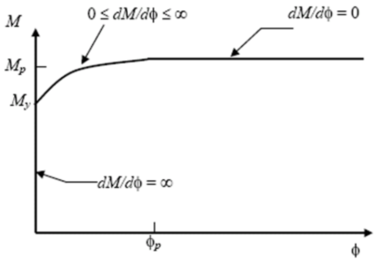

Figure 1: Post-elastic moment-rotation relation for plastic hinges

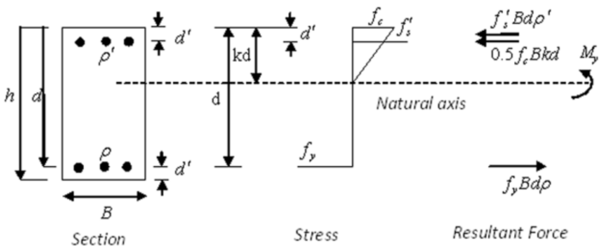

Figure 2: Doubly reinforced member section at first yield

For the case of combined bending moment M and axial force N, the presence of the axial force can be accounted for through the reduction in the moment capacity of the member cross-section due to the following interaction constraint equation with the lower and upper bounds that represent the first-yield and fully-plastic phases, respectively:

$$\frac{1}{f_s} \le \frac{M}{M_P} + \left[\frac{N}{N_P}\right]^a \le 1 \quad , \tag{10}$$

Here $f_s = \frac{M_p}{M_y}$ is the section shape factor, N_P is the fully-plastic axial force capacity, and a is a constant which depends on the section shape (a =1 in this study).

Replacing the connection's rotational stiffness, R, with Rp in equation (3), the ‘plasticity-factor’, p, can be calculated which defines the degradation of the flexural stiffness of a member section with post-elastic behaviour.

By replacing the rigidity-factors, r, with the plasticity-factors, p, in the member stiffness matrices, Kj, the influence of the post-elastic section behaviour on the overall behaviour of a structure under increasing loads can be accounted for through an incremental load pushover analysis similar to a semi-rigid analysis.

MULTI-OBJECTIVE OPTIMIZATION

In many optimization problems, minimizing the structural cost is the most common design objective. Since a complete estimation of a building's real cost requires accurate information that is not readily available, the cost of the members is taken as the cost objective function. Usually, the total frame weight is assumed proportional to its material weight and thus the total construction cost can be interpreted based on the weight of the structure.

For an RC building having n members with rectangular cross sections, where the member dimensions, Bi (width), and Di (depth), are fixed and the topology of a building’s structural system is predefined, the total weight of the reinforcing steel can be given by the formula:

$$f_1(x) = \sum_{j=1}^{n} \upsilon L_j (A_{s,j} + A'_{s,j}) / W_{\max} \quad , \tag{11}$$

Where x is the design variable vector required to be obtained in order to minimize the objective function; n is the number of elements; Lj is the length; $A'_{s,j}$ and $A_{s,j}$ are the top and bottom reinforcement areas of the jth member, respectively; and υ is the material density. The weight function, can be normalized by the maximum possible weight of the frame, (Wmax=894.70 kN).

The response of a structure under earthquake loading shows that the deformation concentration at a weak story leads to the structure's collapse. Therefore, it can be concluded that a uniform interstory drift results in less damage to the building. Thus, the interstory drift is regarded as the primary parameter in evaluating the structural performance.

With the uniform interstory drift, the stories of a building sustain identical damages and thus the structural efficiency improves. To enhance the structural performance, another objective function is defined in terms of the interstory drift at the CP performance level which was defined by (Xu, 1995) as:

$$f_2(x) = \left[(1/ns)\sum_{s=1}^{ns-1}[((v_s^{CP}(x)/\Delta^{CP}(x))(H/H_s)-1]^2 \right]^{\frac{1}{2}}, \tag{12}$$

Where ns is the number of stories; Hs and H are the distances from the building ground level to the story s and the roof, respectively; while $v_s^{CP}(x)$ and $\Delta^{CP}(x)$ are the lateral translations of the story s and the building roof at the CP performance level, respectively.

All the contemporary seismic design procedures are based on the fact that the extent of damage sustained by a structure under earthquake loads depends on the energy absorption capacity of the structure, and that a favourable design allows for the absorption and dissipation of the kinetic energy through inelastic deformation. In this regard, and in order to unify the story energy, another objective function can be written as:

$$f_3(x) = \left[\sum_{i=1}^{ns-1} \left(\frac{\sum_{j=1}^{nf} M_{ij}^{CP}\theta_{ij}^{CP}}{\sum_{j=1}^{nf} M_{nsj}^{CP}\theta_{nsj}^{CP}} - 1 \right)^2 \right]^{\frac{1}{2}}, \tag{13}$$

Where Mij and θ_{ij} are the moment node and nodal rotation in the ith story and the jth vertical row, while nf is the number of spans+1. If the energy distribution in all the stories is the same, the amount of the optimized function f3 is equal to zero.

Applying the three above functions simultaneously in the structural optimization, the general objective function can be expressed as:

$$f(x) = \omega_1 f_1(x) + \omega_2 f_2(x) + \omega_3 f_3(x) = \omega_1 \sum_{j=1}^{n} \upsilon L_j (A_{s,j} + A'_{s,j})/W_{\max}$$
$$+\omega_2 \left[(1/ns)\sum_{s=1}^{ns-1}[((v_s^{CP}(x)/\Delta^{CP}(x))(H/H_s)-1]^2 \right]^{\frac{1}{2}} + \omega_3 \left[\sum_{i=1}^{ns-1}\left[\frac{\sum_{j=1}^{nf} M_{ij}^{CP}\theta_{ij}^{CP}}{\sum_{j=1}^{nf} M_{nsj}^{CP}\theta_{nsj}^{CP}} - 1\right]^2 \right]^{\frac{1}{2}} \tag{14}$$

where ω_1, ω_2 and ω_3 are the combination factors for the function f1, f2 and f3 which together form the general function. To account for the portions of the weight function, f1, the drift function, f2 and energy function, f3, some numerical experiments should be conducted to avoid premature weight convergence.

The steel reinforcement, affects the ductility of RC frames under the inelastic seismic loads, so it is considered as a design variable in the optimization process. Assuming that adequate shear capacity strength is provided for each member, only the longitudinal flexural reinforcement of the member sections is taken as a design variable, and the transverse shear reinforcement is considered as invariant. For simplicity, the compression steel area, A'_s, is assumed to be the same as A_s for beams and columns such that the two steel areas can be reduced to one design variable for each member.

In a design optimization procedure, sizing variables may be continuous or discrete. Here, the design variables are assumed to be continuous between an upper and a lower limit.

DESIGN CONSTRAINTS

In order to control the performance of the structure, constraints are added in the process of optimization as follows:

$$\begin{cases} \delta_s^i(x) \le \bar{\delta}^i \\ \Delta_s^i(x) \le \bar{\Delta}^i \end{cases}, \qquad \text{(i=OP, IO, LS and CP)} \tag{15}$$

Where δs is the interstory drift of the story s (i.e., $\delta_s = v_s - v_{s-1}$, the difference between the drift v_s at the story s and the drift v_{s-1} at the story s-1); $\bar{\delta}$ is the allowable interstory drift; and Δ and $\bar{\Delta}$ are the roof drift and the allowable roof drift, respectively.

In the design process, all the beams and columns are treated as beam-column members and all the inelastic deformations are assumed to occur at the plastic hinges which are located at the ends of each frame member, and also the members are assumed to be fully elastic between the plastic hinges. The lower and upper limits of 0.5% and 2% are imposed on the values of the design variables for the tension reinforcement steel ratios, based on the concrete building codes.

EXAMPLE: FIVE STORY

According to (FEMA-273, 1997), the base shear in each performance level is:

$$V_b^i = W(S_a^i / g), \qquad \text{(i=OP, IO, LS and CP)} \tag{16}$$

Where W is the seismic weight of the structure; g is the gravitational acceleration; and S_a^i is the spectral acceleration for each performance level that can be expressed as in (FEMA-273, 1997),

$$S_a^i = \begin{cases} F_a^i S_s^i (0.4 + 3T/T_0^i) & 0 < T \le 0.2T_0^i \\ F_a^i S_s^i & 0.2T_0^i < T \le T_0^i \\ F_v^i S_1^i / T & T > T_0^i \end{cases} \tag{17}$$

Here T is the elastic period of the structure; T0i is the period at which the constant acceleration and constant velocity regions of the response spectrum intersect for the design earthquake associated with the performance level i; and S_s^i and S_1^i are the corresponding short-period and one-second period response spectrum parameters, respectively.

The concrete moment frame which is considered in this example, is a five-story, five-bay frame as shown in figure 3. The framework consists of 55 members and 20 design variables, A1-A20. According to (FEMA-356, 2000), the allowable interstory drifts, $\bar{\delta}$, are taken as 0.01h, 0.02h and 0.04h for the IO, LS and CP levels, respectively. Also, the allowable roof drifts, $\bar{\Delta}$, for the IO, LS and CP levels are assumed as 0.01H, 0.02H and 0.04H, respectively. h and H are the heights from the base of the building to each story and the roof, respectively.

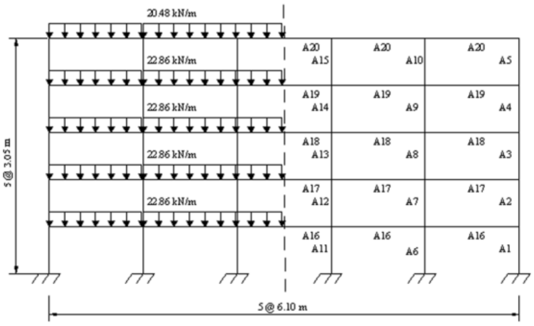

Figure 3: Example: five-story five-bay moment frame

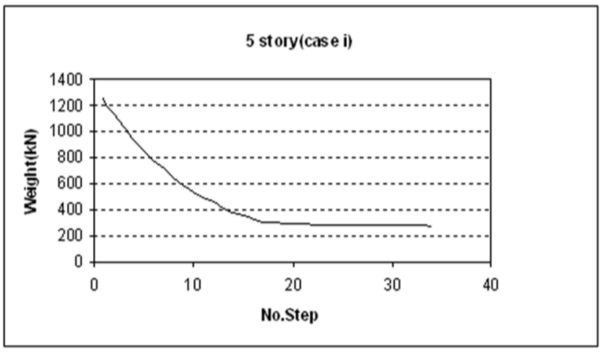

Figure 4: Weight evolution during optimization process.

DISCUSSION AND CONCLUSIONS

The results here are based on the three assumed values for ω_1, ω_2 and ω_3 as follows:
i) ω_1=1, ω_2=0, ω_3=0, ii) ω_1=0.95, ω_2=0.05, ω_3=0, iii) ω_1=0.94, ω_2=0, ω_3=0.06.
The optimized cross sections and weights in the three above cases are presented in table 1. Figure 4 shows the weight evolution during the optimization process for case i. As is shown in figures 5, the drift distributions along the frame height for the three cases (i, ii, iii), as related to the OP, IO and LS levels are alike.

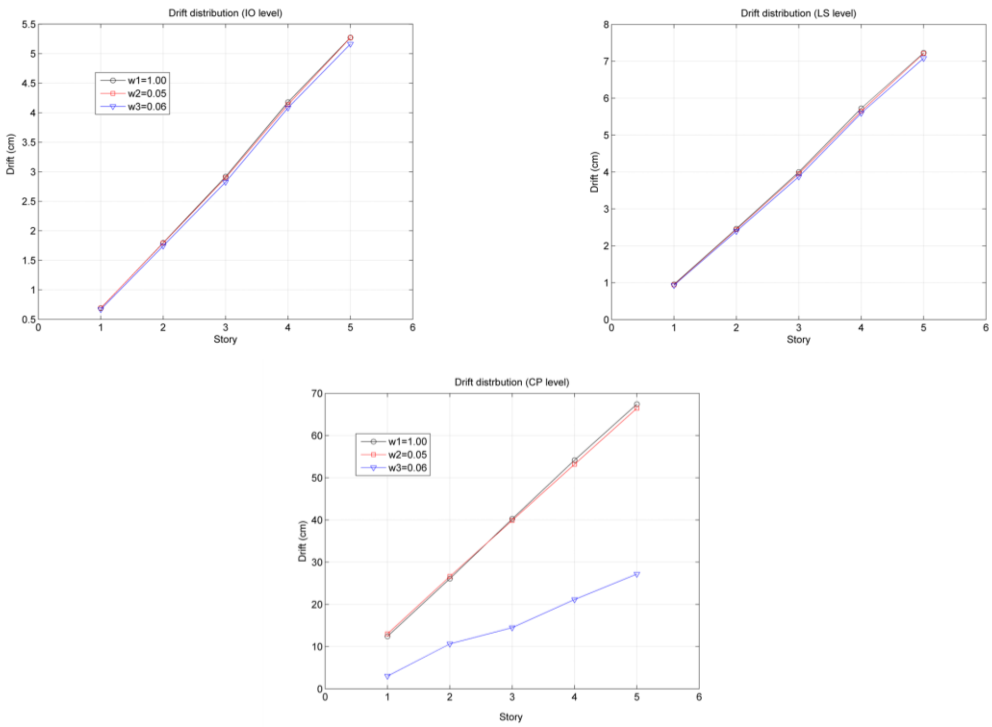

Figure 5: drift distribution in IO, LS and CP.

However, since f_2 and f_3 are defined for the CP level, there are some differences between drift distribution results for the three mentioned cases. In order to evaluate the damage sustained for each case, a damage index has been defined as:

$$DMI = \frac{\sum_{i=1}^{PH}(1-P_i)}{PH}, \tag{18}$$

Where PH is the number of hinges, and P_i is the plasticity factor of the i^{th} hinge. To find the best case, an efficiency factor can be expressed as:

$$EF_i = 100\frac{1-DMI}{W_{opt,i}}, \tag{19}$$

Where $W_{opt,i}$ is the optimum weight of the i^{th} case. In table 1 the efficiency factor for the three cases are compared. As the results show, the maximum value of EF_i is for the case iii in which the energy function is included.

Table 1: Optimum weight, efficiency factor and last drift for five story frame

	i	ii	iii
Weight (kN)	271.7594	271.995	289.7537
Efficiency factor	0.200565	0.296299	0.439899
Last drift (cm)	-3.8054786	-4.1296816	-3.6668146

For comparison with the real behavior, nonlinear dynamic analyses for the optimized frames with elements of distributed inelasticity are performed for fifty year period earthquakes, with 2% intensity (Erzincan, Turkey record) and the results are shown in figure 6.

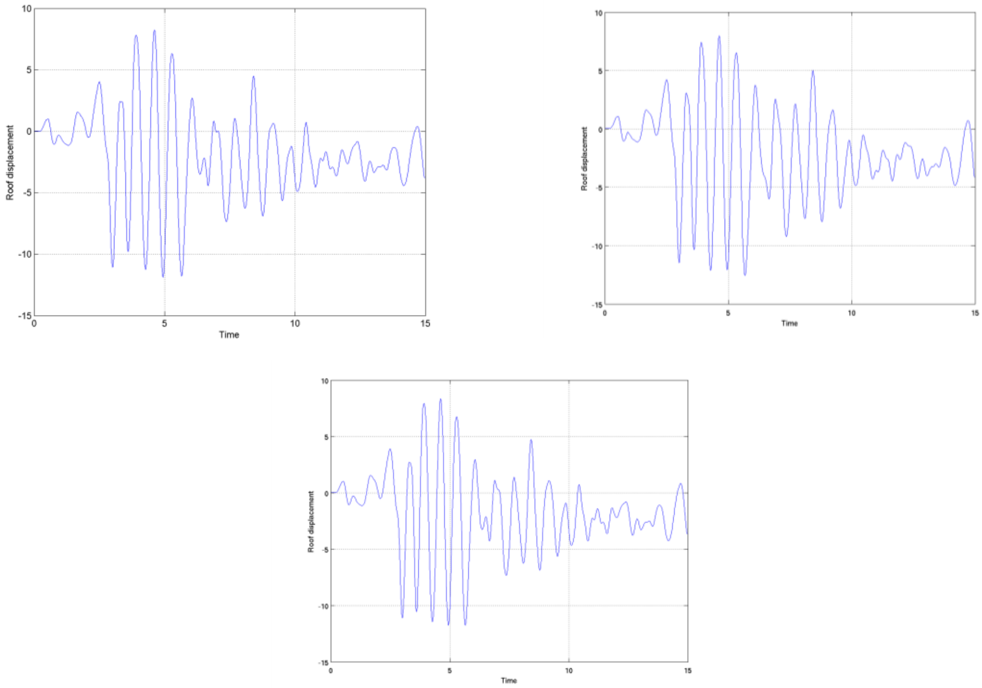

Figure 6: Time history of horizontal roof displacement under nonlinear transient response with distributed inelasticity element (cm), (a): case i, (b): case ii, (c): case iii.

The last drifts (the residual drifts after the ground motion stops) in the three cases have been compared in table 1. Since there are some constraints on the steel reinforcement areas and the story drifts in accordance with the current codes, the differences among the results for various performance levels are not as high.

As is shown, the optimization process on the basis of the energy function, f_3, leads to a decrease of pulse intensity and the last drift and therefore, moderates the frame response. This case verifies that minor damage occurs in these frames compared with the optimized frames in cases *i* and *ii*.

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