MODAL ANALYSIS FOR FLEXIBLE BASE ECCENTRIC BUILDINGS

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ABSTRACT

The modal translation and modal rotation are not the same even at elastic state for the systems with the non-classical damping. The 2DOF modal stick based on the 2DOF modal equation has been developed to deal with the seismic analysis of one-way asymmetric fixed base systems and allows the modal rotation to differ from the modal translation. In this paper the applicability of this method in the seismic linear analysis of flexible base asymmetric buildings has been studied. The results show that the efficiency of this modal method using first limited number of modes (usually three fist modes) for the structure response is very good; but for the foundation response seems that it is necessary to consider enough number of modes because of the effects of higher modes of SSI system. Also the results show that generally the estimation of this method for the displacement is better than rotation estimation.

INTRODUCTION

Considering the SSI effects, Includes the non-classical damping and frequency-dependent interaction forces. In order to encounter the non-classical damping, equivalent modal damping was calculated by quantifying the dissipated energy (Novak and Hifnawy, 1983) that was not easily to apply in a computer program. To consider the frequency-dependent interaction forces, the SSI problem was solved in the frequency domain. However, frequency domain analysis is only useful for linear responses and is not practical for engineering structures (Wolf, 1985).

Goel (2001) shows that neglecting the off-diagonal terms of the transformed damping matrix for a non-classically damped one-way fixed based asymmetric structures, may lead to Significant errors. Lin and Tsai (2007a) have studied the effectiveness of the modal analysis using two-degree-of-freedom (2DOF) modal stick based on the 2DOF modal equation (Lin and Tsai, 2007b) to deal with the seismic analysis of one-way asymmetric elastic systems with supplemental damping. This 2DOF modal stick allows that the modal translation and modal rotation to not the same even at elastic state. Comparison of this method with the exact solution and the conventional approximate method, which neglects the off-diagonal elements in the transformed damping matrix for the one-storey and three-storey buildings has lead to good results. They also investigate the 3DOF modal equation (Lin and Tsai, 2008a) for the seismic analysis of two-way asymmetric elastic systems with supplemental damping (Lin and Tsai, 2008b) and achieve to the acceptable approximation.

Lin and Tsai (2008b) proposed a new modal response history analysis procedure for the asymmetric linear soil–structure systems based on the mentioned MDOF (2DOF and 3DOF) modal equation. In this paper after the review of mentioned modal solution, the applicability of this method in the seismic linear analysis of asymmetric steel braced frames has been studied.

FLEXIBLE BASE ASYMMETRIC STRUCTURE

For a one-way asymmetric N-story building resting on a one-way asymmetric foundation, 2N+3 degree of freedom must be considered. Because of the system is symmetric about Y axe, the equation of motion about the X axe and rotation about the vertical axe (Z) are couple. Therefore for each story, tow degree of freedom is considered (translation in the X direction and rotation about Z axe). Also 3 degree of freedom has been appropriated for the translation, rotation and rocking of foundation. Therefore the equation of motion for SSI system is:

$$\mathbf{M}^{*}\ddot{\mathbf{W}} + \mathbf{C}^{*}\dot{\mathbf{W}} + \mathbf{K}^{*}\mathbf{W} = -\mathbf{P}^{*} \tag{1}$$

where

$$\mathbf{W} = \begin{bmatrix} \mathbf{U}'_{jc} \\ \boldsymbol{\theta}^{t}_{j} \\ U_{0} \\ -_{0} \\ \cdot_{0} \end{bmatrix}_{(2N+3)\times 1}, \quad \mathbf{P}^{*} = \begin{bmatrix} \mathbf{m}\,\mathbf{1}\ddot{u}_{gx} \\ \mathbf{0} \\ m_{0}\ddot{u}_{gx} \\ 0 \\ 0 \end{bmatrix}_{(2N+3)\times 1} \tag{2a}$$

$$\mathbf{M}^{*} = \begin{bmatrix} \mathbf{m} & & & & \\ & r^{2}\mathbf{m} & & & \\ & & m_{0} & & \\ & & & r^{2}m_{0} & \\ & & & & \sum_{j=0}^{N} I_{yj} \end{bmatrix}_{(2N+3)\times(2N+3)} \tag{2b}$$

$$\mathbf{C}^{*} = \begin{bmatrix} \mathbf{C}_{x} & f\,\mathbf{C}_{x} & -\mathbf{C}_{x}\mathbf{1} & -\mathbf{C}_{x}\mathbf{f} & -\mathbf{C}_{x}\mathbf{h} \\ & \mathbf{C}_{-} & -\mathbf{C}_{x}\mathbf{f} & -\mathbf{C}_{-}\mathbf{1} & -f\,\mathbf{C}_{x}\mathbf{h} \\ & & C_{T} + \mathbf{1}^{T}\mathbf{C}_{x}\mathbf{1} & \mathbf{f}^{T}\mathbf{C}_{x}\mathbf{1} & \mathbf{1}^{T}\mathbf{C}_{x}\mathbf{h} \\ & symm. & & C_{z} + \mathbf{1}^{T}\mathbf{C}_{-}\mathbf{1} & \mathbf{f}^{T}\mathbf{C}_{x}\mathbf{h} \\ & & & & C_{r} + \mathbf{h}^{T}\mathbf{C}_{x}\mathbf{h} \end{bmatrix} \tag{3}$$

$$\mathbf{K}^{*} = \begin{bmatrix} \mathbf{K}_{x} & f\,\mathbf{K}_{x} & -\mathbf{K}_{x}\mathbf{1} & -\mathbf{K}_{x}\mathbf{f} & -\mathbf{K}_{x}\mathbf{h} \\ & \mathbf{K}_{-} & -\mathbf{K}_{x}\mathbf{f} & -\mathbf{K}_{-}\mathbf{1} & -f\,\mathbf{K}_{x}\mathbf{h} \\ & & K_{T} + \mathbf{1}^{T}\mathbf{K}_{x}\mathbf{1} & \mathbf{f}^{T}\mathbf{K}_{x}\mathbf{1} & \mathbf{1}^{T}\mathbf{K}_{x}\mathbf{h} \\ & symm. & & K_{z} + \mathbf{1}^{T}\mathbf{K}_{-}\mathbf{1} & \mathbf{f}^{T}\mathbf{K}_{x}\mathbf{h} \\ & & & & K_{r} + \mathbf{h}^{T}\mathbf{K}_{x}\mathbf{h} \end{bmatrix} \tag{4}$$

The m shown in the previous equation is an $N \times N$ diagonal matrix composed of each floor mass. In these equations, f is eccentricity measured from CR to CM along the Y axe and **f** is a $N \times 1$ column vector with all elements equal to f . Also **h** is a $N \times 1$ column vector composed of the story heights measured from the ground level to each floor; the **0** and **1** are the column vectors whose elements are equal to zero and one, respectively. On the other hand m_0 is mass of foundation; r is radius of gyration of any floor deck about CM;

I_{yj} is moments of inertia of the jth floor about the axe through the CM and parallel to the Y axe respectively; $\ddot{u}_{gx}$ is ground acceleration record along the X axe, respectively; $\mathbf{U}'_{jc}$ and $\boldsymbol{\theta}^t_j$ are degrees of freedom of the superstructure as equation (5); and finally, $U_0, \bar{\ }_0, \cdot_0$ are degrees of freedom at the foundation associated with translations, twist and rocking.

$$\begin{aligned} \mathbf{U}'_{jc} &= U_0\mathbf{1} + \text{¤}_0\mathbf{h} + \mathbf{U}_{jc} \\ \boldsymbol{\theta}^t_j &= \bar{\ }_0\mathbf{1} + \boldsymbol{\theta}_{jc} \end{aligned} \tag{5}$$

It must be noted that

$$\mathbf{K}_- = \mathbf{K}_{-R} + f^2\mathbf{K}_x \tag{6}$$

and $\mathbf{K}_{-R}$, $\mathbf{K}_x$ are defined about the CR. Also K_T, K_z and K_r that are translational, rotational and rocking stiffness of foundation and their corresponding damping calculated from the impedance functions proposed by Richart et al. (1970) as the frequency independent functions.

MODAL EQUATION OF MOTION

According to the Modal Pushover Analysis (MPA) basic equation (Chopra and Goel, 2004), $\mathbf{P}^*$ in Eq.(1) can be rewritten as:

$$\mathbf{P}^* = \sum_{n=1}^{2N+3} \mathbf{s}_n \Gamma_{xn}\ddot{u}_{gx} = \sum_{n=1}^{2N+3} \mathbf{M}^*\boldsymbol{\varphi}_n \Gamma_{xn}\ddot{u}_{gx} \tag{7}$$

that $\boldsymbol{\varphi}_n$ is undamped mode shape obtained from $\mathbf{K}^*$ and $\mathbf{M}^*$; and Γ_{xn} is modal participation factors defined as:

$$\Gamma_{xn} = \frac{\boldsymbol{\varphi}_n^T\mathbf{M}^*[\mathbf{1}^T \quad \mathbf{0}^T \quad 1 \quad 0 \quad 0]^T}{\boldsymbol{\varphi}_n^T\mathbf{M}^*\boldsymbol{\varphi}_n} \tag{8}$$

and $\mathbf{s}_n$ is nth modal inertial force distribution equivalent to $\mathbf{M}^*\boldsymbol{\varphi}_n$. Because of the system is assumed elastic, under the nth modal inertial force only the nth undamped modal displacement response, $\mathbf{W}_n$, will be excited. Therefore, Eq. (1) can be written as:

$$\mathbf{M}^*\ddot{\mathbf{W}}_n + \mathbf{C}^*\dot{\mathbf{W}}_n + \mathbf{K}^*\mathbf{W}_n = -\mathbf{s}_n\Gamma_{xn}\ddot{u}_{gx} \tag{9}$$

where

$$\mathbf{W}_n = \begin{bmatrix} \boldsymbol{\varphi}_{\mathbf{un}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & \boldsymbol{\varphi}_{\boldsymbol{\theta}\mathbf{n}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & & \text{¤}_{u_0 n} & 0 & 0 \\ & & & \text{¤}_{-_0 n} & 0 \\ & & & & \text{¤}_{\text{¤}_0 n} \end{bmatrix}_{(2N+3)\times 5} \begin{bmatrix} D_{un} \\ D_{-n} \\ D_{u_0 n} \\ D_{-_0 n} \\ D_{\text{¤}_0 n} \end{bmatrix}_{5\times 1} = \mathbf{T}_n\mathbf{D}_n \tag{10}$$

Because of the nonclassical damping of SSI system, the elements of the $\mathbf{D}_n$ are not the same (Lin and Tsai, 2007a) . Substituting Eq. (10) into Eq. (9) and premultiplying both sides of Eq. (9) by $\mathbf{T}_n^T$, leads to:

$$\mathbf{M}_n\ddot{\mathbf{D}}_n + \mathbf{C}_n\dot{\mathbf{D}}_n + \mathbf{K}_n\mathbf{D}_n = -\mathbf{M}_n\boldsymbol{\iota}\Gamma_{xn}\ddot{u}_{gx} \tag{11}$$

where

$$\mathbf{M}_n = \mathbf{T}_n^T \mathbf{M}^* \mathbf{T}_n \quad , \quad \mathbf{C}_n = \mathbf{T}_n^T \mathbf{C}^* \mathbf{T}_n \quad , \quad \mathbf{K}_n = \mathbf{T}_n^T \mathbf{K}^* \mathbf{T}_n \tag{12}$$

in which M_n , C_n , K_n are 5×5 matrices and $\boldsymbol{\iota}$ is a 5×1 column vector with all elements equal to one. Therefore for every mode of the super structure, a set of five couple equations must be solved Instead of a set of 2N+3 couple equations. Thus, the total response histories of the original SSI system were calculated as:

$$\mathbf{W}(t) = \sum_{n=1}^{2N+3} \mathbf{W}_n(t) \approx \sum_{n=1}^{2N+3} \mathbf{T}_n \mathbf{D}_n(t) \tag{13}$$

A MATLAB code based on this modal method is developed in this paper. It will be continued by the results of this code for a 5- story steel braced frame with SSI effects.

ASYMMETRIC EXAMPLE STRUCTURE AND SELECTED GROUND MOTION

The asymmetric structure is a 5-story steel braced frame that story height is 3.5 m for all stories (Fig. 1). All stories is assumed rigid diaphragm; The structure plan is rectangular ($10m \times 16m$) and the excitation is applied in the X direction (parallel to the smaller dimension). The stiffness of structure is symmetric. The beam-column connections and the column base connections are moment released. The frames sections of the of all stories are presented in the Table (1). The building eccentricity is caused by the mass eccentricity equal to 0.25 for the (f / r) ratio in all stories (r is radius of gyration about the center of mass).The mass of each story is 25,000 kg; The ratio of the base mass to the floor mass of the building is 3.0 and the radius, r_o , of the base mass is taken as the radius of a circle having the same area to that of the plan of the building. The damping matrix of the superstructure is stiffness proportional damping, which was determined by setting the damping ratio of the fundamental mode to 2%. The density of the soil medium was taken to be 1,920 kg/m3 and Poisson's ratio of the soil medium was assumed to be 1/3. The shear wave velocities of the soil, Vs, is 100 m/ s.

5-story steel braced frame was subjected to the N-S component of 1940 El Centro Earthquake excitation (Fig. 2) along the X axe.

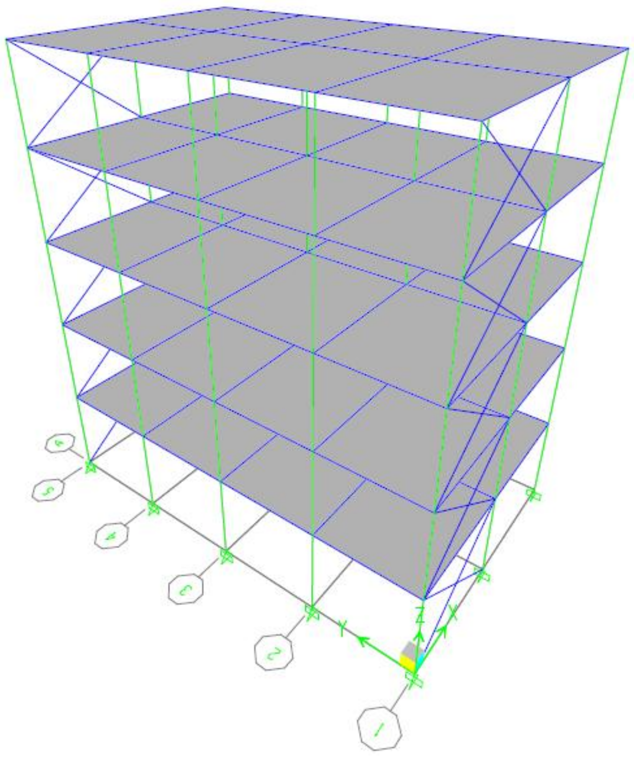

Figure 1. 3D view of asymmetric example structure

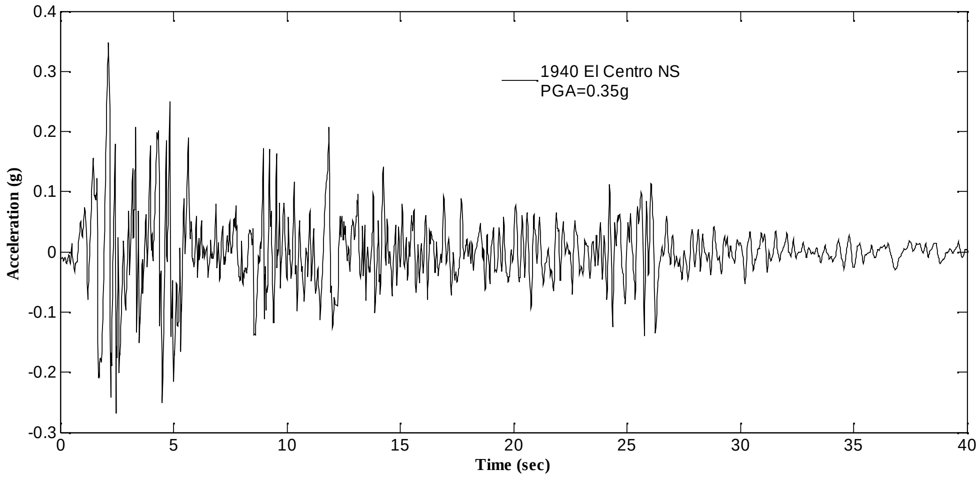

Figure 2. Ground acceleration records for the NS component of the 1940 El Centro Earthquake

Table 1. Frames sections for each stories

Story number	1	2	3	4	5
Bracing section	IPBL 180	IPBL 160	IPBL 140	IPBL 120	IPBL 100
Column section	IPB 300	IPB 280	IPB 260	IPB 240	IPB 220
Beam section	IPE 270	IPE 240	IPE 220	IPE 200	IPE 180

NUMERICAL RESULTS

Numerical results will be presented in the form of CM (Centre of Mass) displacement and story diaphragm rotation of the roof and the foundation. The results from the mentioned modal method (using different number of modes) is compared with the exact solution. The exact solution can be obtained by two approaches. The first is the solving of this modal MDOF using all of modes and the second is the solving of original SSI system by direct integration method.

Because of the system is linear, different subroutine for the equation solving as Beta-New mark or Central Difference lead to the same results. Therefore the Central Difference is has been used in the MATLAB code.

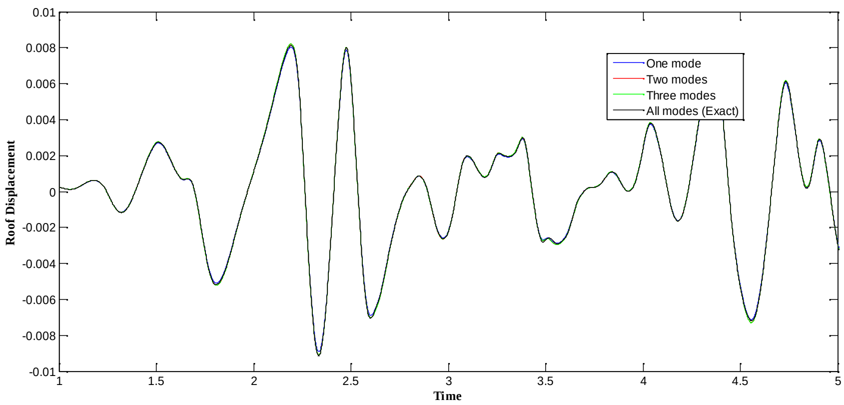

Figure 3. Roof Displacement time history using one, two and three first modes and all modes

The time history of the roof displacement using one, two and three first modes and all modes of the flexible base asymmetric structure is presented in the figure (3). As this figure shows, the results using the only first mode is very near the exact response (by thirteen modes). The time history of the roof rotation using one, two, three and all modes is presented in the figure (4). However the results of displacement are better for a limited number of modes, but the results of rotation are good and its approximation is useful in the practical engineering aims.

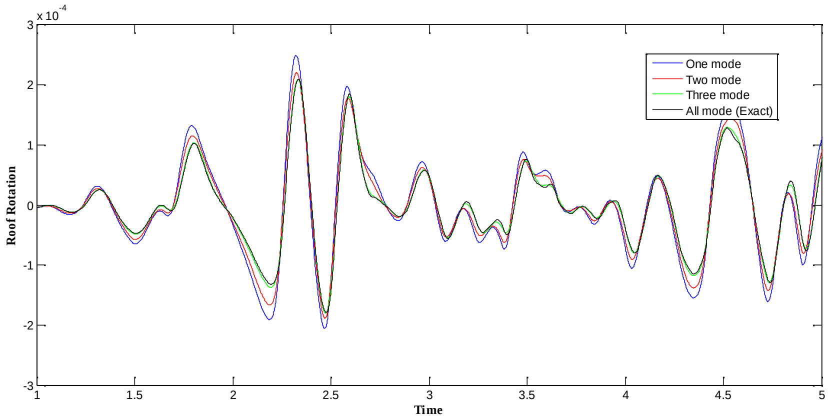

Figure 4. Roof rotation time history by one, two and three first modes and all modes

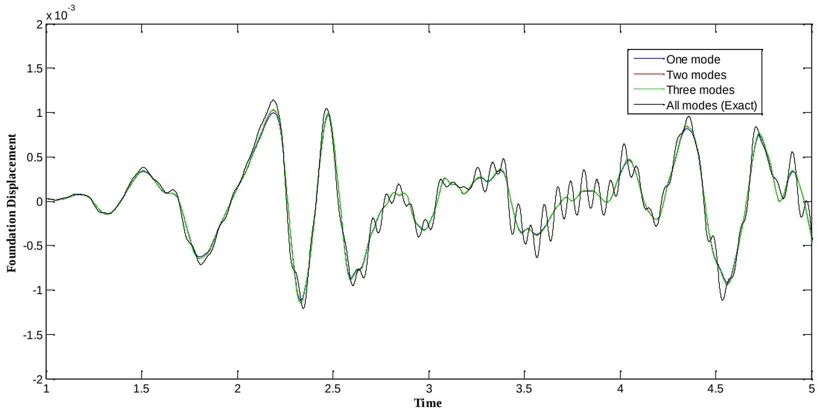

Figure 5. Foundation displacement time history by one, two and three first modes and all modes

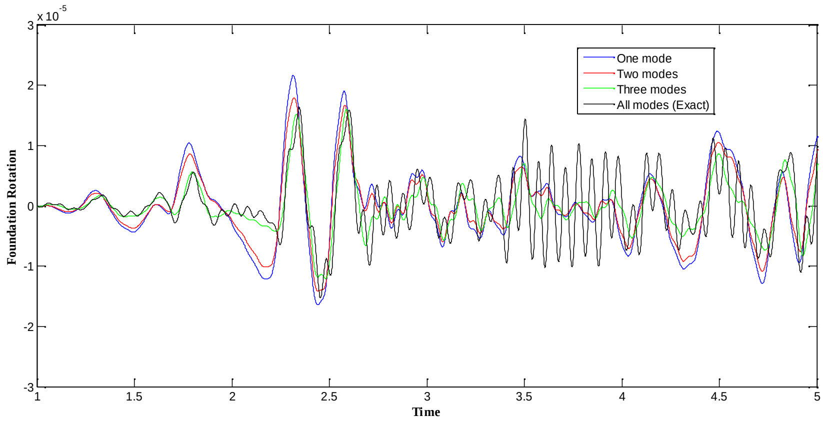

Figure 6. Foundation rotation time history by one, two, three and all modes

The time history of the foundation displacement and its rotation using one, two and three first modes and all modes of the flexible base asymmetric structure is presented in the figure (5) and (6). As the figure (5) shows, the peak response using only first mode is near the exact response; but the total history is not consistent acceptable (even using three first modes). As the figure (6) shows, not only the total history is not acceptable using a limited number of modes, but also some weaker peaks are missed.

The main reason of the weak performance of the using first modes in the estimation of total history of foundation is the effects of higher modes of SSI system and their contribution in the foundation motion.

CONCLUSIONS

MDOF modal equations of motion has been solved instead of the original SSI system, is a powerful and applicable method to deal with a nonclassical damping system (as SSI) when compared with the calculation of the complicated equivalent modal damping. However the only limitation of this method that the SSI system is assumed frequency independent, but it is a acceptable assumption for practical aims. This method allow engineer to deal with a SSI system similar to the conventional modal analysis for a fixed base structure without concerning with equivalent damping or complex modal analysis.

It must be noted that the efficiency of this modal method using first limited number of modes (usually three fist modes) for the structure response is very good; but for the foundation response seems that it is necessary to consider enough number of modes because of the effects of higher modes of SSI system. Also the results show that generally the estimation of this method for the displacement is better than rotation estimation.

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