STIFFNESS MATRIX OF SHEAR-TYPE BUILDINGS EXTRACTED FROM OUTPUT-ONLY DATA

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ABSTRACT

Direct extracting matrices of a building is of major interest of a practical engineers. In this paper a method based on the stochastich subspace state-space identification has been proposed to identify stiffness matrix of structures.Only the output data is used. Once the state matrix of a system is identified by using of a similarity transformation it transforms from discrete time general form to the continuous time classical form of mass-spring-damper systems. Then, with known mass matrix the stiffness and damping matrices are simply in hand by multiplying each corresponding block by mass matrix.

The proposed method is applied on a three-story two-dimansional shear frame. Once a model has been identified, a seismic assessment under Tabas earthquake excitation are examined and their responses are compared with the concern original model responses. The results show that this method is capable to be an appropriate tool for extracting main matrices of structures.

INTRODUCTION

The practical evaluation methods for evaluating dynamic systems like as vibrating structures has been subjected of intense interest in the last decades. Among these methods there are those based on the analysis of structural dynamic response measurements to identify a proper mathematical model corresponding to the state of structure so call system identification methods. So many studies has been conducted in nondestructive evaluation (NDE), active control of systems and structural health monitoring (SHM) relying on these methods. Active control has four key steps: (1) modelling, (2) system identification, (3) design of contollers and observers, and (4) verification. System identification methods as common step of all active control processes play a crucial role(Juang, 1994). Almost in structural health monitoring system identification has undeniable role. SHM aims to give, at every moment during the life of a structure, a diagnosis of the “state” of the constituent materials, of the different parts, and of the full assembly of these parts constituting the structure as a whole. The state of the structure must remain in the domain specified in the design, although this can be altered by normal aging due to usage, by the action of the environment, and by accidental events. Thanks to the time-dimension of monitoring, which makes it possible to consider the full history database of the structure, and with the help of usage monitoring, it can also provide a prognosis evolution of damage, residual life ,etc. (Balageas, et all, 2010). There are many description of SHM in literature, but in all of them system identification has significant place. Structural damage detection through SHM is indebted to system identification development.

The stochastic subspace state space identification (SSI) is known as most powerful output-only method in time domain. In contrast with classic approach this method, which is first introduced by Van Overschee and De Moor in 1996, identifies systems by simple and fast parametrization and non-itterative covergence. Time domain methods are usually consummated as a optimization that results a need to initial informations about the system to start the process. But since the SSI construct the block Hankel matrix from Markov parameters then it doesn't need to these data because the markov parameters can be determined using input/output data. In contrast with classic methods in time domain the SSI doesn't need to initial model parametrization and the calculations pursue by limited number predetermined steps. These steps contain a set of algebraic operations such as QR and singular value decomposition (Van Overschee, De Moor, 1996). Peeters applied this method in his thesis on civil structures (Peeters, 2000). Almost Brinker and Anderson tried to explain employed mathematic consepts in the SSI by a much simple manner (Brinker, Anderson, 2006).

In this paper a method is proposed based on well known SSI algorithm to identify the structural stiffness and damping matrices by using of the known mass matrix. Only the output responses of a system need to be measured. The proposed method is applied in three two dimensional multistory frames and the identified properties are compared with corresponding original model. Then, the identified models and their original model are excited by Tabas earthquake excitation. It is shown that the proposed method give a suitable tool for linear seismic evaluation of structures.

STOCHASTIC STATE-SPACE MODEL

As it is mentioned in the previous section, taking into account the effects of process and measurement noises, which are respectively shown by w_k and v_k, in both the state-space and observation equations of motion, the state-space discrete time-domain model can be expressed as a general form as follows

$$\begin{aligned} x_{k+1} &= A_d x_k + B_d u_k + w_k \\ y_k &= C x_k + D u_k + v_k \end{aligned} \tag{1}$$

Both w_k and v_k are unmeasurable vector signals, and are assumed that they are zero mean, independent weak stationary with uniform distribution random signals. It is assumed that

$$E\begin{bmatrix} w(t) \\ v(t) \end{bmatrix} = 0\,,\, E\left[\begin{bmatrix} w(t) \\ v(t) \end{bmatrix}\begin{bmatrix} w^T(j) & v^T(j) \end{bmatrix}\right] = \begin{bmatrix} \boldsymbol{Q} & \boldsymbol{S} \\ \boldsymbol{S^T} & \boldsymbol{R} \end{bmatrix}\delta(t-j) \tag{2}$$

in which E denotes the expected value operator and δ the Kroncker delta function. Q, S and R are the covariance matrices of the noise sequences wk and vk. In general form there are many difficulty to measure all external disturbances especially when excitation sources are various and/or its amplitude is very small. In these cases it is preferred that only system outputs to be measured (e.g. ambient vibration tests), so Eq (1) is modified as:

$$\begin{aligned} x_{k+1} &= A_d x_k + w_k \\ y_k &= C x_k + v_k \end{aligned} \tag{3}$$

In this case, each various sources of excitation (inputs) or disturbutions are substituted as one stochastic term as a noise signal. Identified state matrix "A" has a general form as follow:

$$A = \begin{bmatrix} \times & \times & \dots & \times \\ \times & \times & \dots & \times \\ \vdots & \vdots & \ddots & \vdots \\ \times & \times & \dots & \times \end{bmatrix} \tag{4}$$

where A is a matrix full of numbers between zero and one. If the state matrix in general form change into the known classical form as follow:

$$A_c = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \tag{5}$$

With known mass matrix, the stiffness and damping matrices can be calculated simply. Let the quadruple (A, B, C, D) be a minimal realization that if transform to continuous time-domain form the classical form of state matrix "A" will appear. Every minimal realization can transform to this quadruple with a similarity transformation. Then if the minimal realization is in hand, it should exist a transformation that transforms the second to the first. By placing equal eigenvalues of two realization we have:

$$\Lambda = \tilde{\Lambda} \tag{6}$$

where Λ and $\tilde{\Lambda}$ are the eigenvalues of A and $\tilde{A}$ (the minimal realization of A), respectively. Rewriting the equation by eigenvectors gives:

$$\psi^{-1}A\psi = \tilde{\psi}^{-1}\tilde{A}\tilde{\psi} \tag{7}$$

where ψ and $\tilde{\psi}$ are the eigenvectors of A and $\tilde{A}$ respectively. Multiplying the equation by ψ from left and ψ^{-1} from right gives:

$$A = \psi\tilde{\psi}^{-1}\tilde{A}\tilde{\psi}\psi^{-1} \tag{8}$$

Let transform matrix be $T = \tilde{\psi}\psi^{-1}$ then A can be calculated by knowing $\tilde{A}$ and T as $A = T^{-1}\tilde{A}T$ (note that A is the discrete time-domain state matrix that if transform to continuous time-domain the classical state matrix will appear). Since the quadruple (A, B, C, D) is unknown, it should be a relation between ψ and $\tilde{\psi}$ to constructing the transform matrix T. It can be shown that ψ can be calculated by given $\tilde{\psi}$, $\tilde{C}$ and $\tilde{\Lambda}$ as follow:

$$\psi = \begin{bmatrix} \tilde{C}\tilde{\psi} \\ \tilde{C}\tilde{\psi}\tilde{\Lambda} \end{bmatrix} \tag{9}$$

Now the transform matrix T can be constructed by given ψ and $\tilde{\psi}$.So any minimal realization of the system can transform to the classical form. The stiffness and damping matriced of system will be calculated simply by multiplying $-M^{-1}K$ block and $-M^{-1}C$ block by mass matrix.

NUMERICAL EXAMPLE

The proposed method has been applied on three numerical models. The example is a three story 2D shear building frames. The shear frames have one transational degree of freedom for each story. The damping matrix is considered classical form and the damping ratio 0.05 for the first and third modes. The input exitation is normal distribution Guassian zero mean white noise with standard deviation equal to 1 that generated with time steps equal to 0.01s. The models have been excited by the noise then a time history analysis has been perform to acquire the dynamic responses. Acquired absolute acceleration responses is employed in proposed algorithm and the results are evaluated. Then the identified models are excited again, this time by Tabas earthquake excitation. Different responses of the excited identified models as displacements, drifts and others are compared with corresponding original model responses in the read more.

The identified frequencies and damping ratios of three story shear model are shown in table.1:

Table 1. Dynamic properties of three story shear model

Property	Identified	Numerical model
Frequency	4.15	4.19
	9.40	9.42
	17.09	17.13
Damping Ratio	5.43	5
	3.76	4.1
	5.14	5

One can clearly be seen is good accuracy in identified frequencies and damping ratios with maximum relative error percent equal to 0.95% and 8.6% repectively. The one story shear and general model that their results have been refused of bringing have good results like as this one. The identified mode shapes are shown in Figure.1

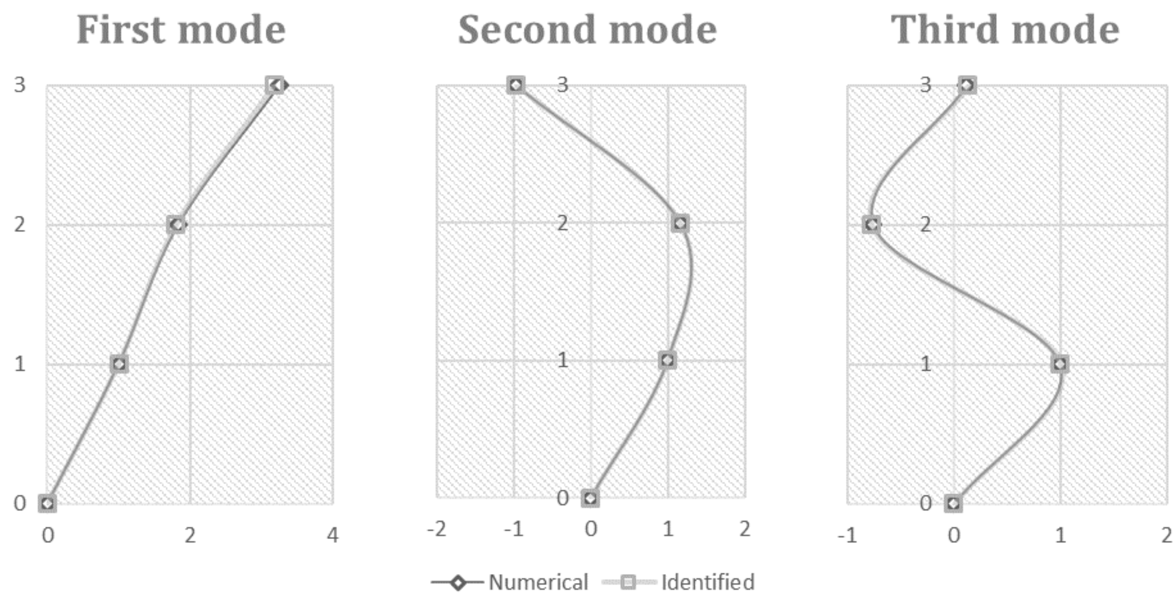

Figure 1. The mode shapes of three story shear model

The identified state matrix in discrete time domain for this model by noise free data is shown below:

$$A_d = \begin{bmatrix} 0.82 & 0.52 & -0.06 & 0.15 & 0.17 & -0.05 \\ -0.57 & 0.77 & -0.08 & 0.18 & 0.20 & -0.07 \\ -0.01 & -0.09 & 0.88 & 0.30 & 0.34 & -0.11 \\ -0.02 & -0.23 & -0.29 & 0.92 & -0.10 & -0.00 \\ -0.02 & -0.23 & -0.29 & -0.09 & 0.53 & -0.71 \\ -0.02 & -0.14 & -0.14 & -0.02 & 0.66 & 0.52 \end{bmatrix} \tag{10}$$

Transforming it in to classical form in continuous time domain gives:

$$A_c = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -8.28e+03 & 4.15e+03 & 31.572 & -10.94 & 6.89 & 0.40 \\ 4.22e+03 & -5.81e+03 & 1.59e+03 - & -2.52 & -3.97 & 0.76 \\ 50.90 & 1.54e03 & -1.58e+03 & -4.92 & 3.69 & -3.51 \end{bmatrix} \tag{11}$$

Then multiplying each matrix blocks on mass matrix give the stiffness and damping matrices. The identified and numerical stiffness matrices of this model are represented in table.2. As can be seen the identified stiffness matrix is not completely symmetric, so it is symmetrized by averaging between off diagonal arrays once to evaluate the seismic responses of identified model.

Table 2. Stiffness matrix of three story shear model (kN/m)

<table>
<tr><th>Model</th><th>Identified matrix</th><th>Identified and symmetrized matrix</th></tr>
<tr><td>Numerical</td><td colspan="2">$\begin{bmatrix} 1.68E+05 & -8.40E+04 & 0.00E+00 \\ -8.40E+04 & 1.16E+05 & -3.17E+04 \\ 0.00E+00 & -3.17E+04 & 3.17E+04 \end{bmatrix}$</td></tr>
<tr><td>Identified</td><td>$\begin{bmatrix} 1.66E+05 & -8.34E+04 & -5.66E+02 \\ -8.42E+04 & 1.16E+05 & -3.17E+04 \\ 1.54E+03 & -3.28E+04 & 3.18E+04 \end{bmatrix}$</td><td>$\begin{bmatrix} 1.66E+05 & -8.38E+04 & 4.88E+02 \\ -8.38E+04 & 1.16E+05 & -3.23E+04 \\ 4.88E+02 & -3.23E+04 & 3.18E+04 \end{bmatrix}$</td></tr>
</table>

Once a model has been identified it is excited by Tabas earthquake excitation. The displacement response of the third story of current model is represented below:

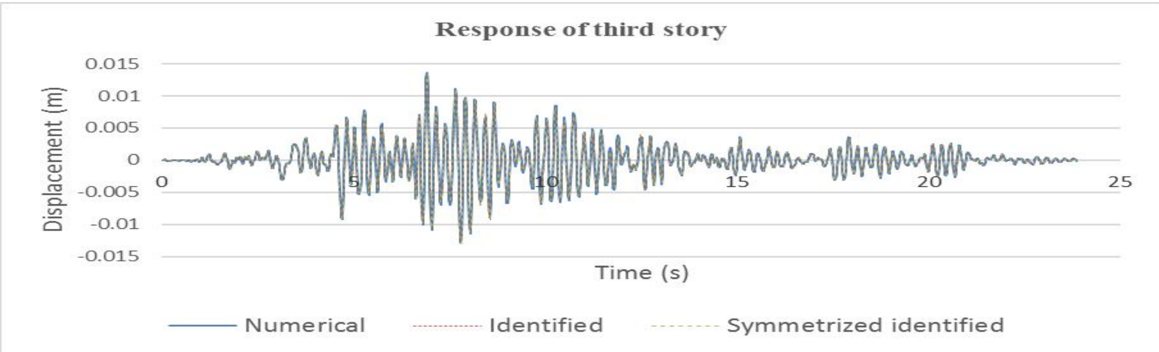

Figure 2. Displacement response of three story shear model

As can be seen there is a good compliance between identified and original models responses. The numerical investigation also confirms this claim. The maximum displacements of current model stories are shown in the table.3:

Table 3. Maximmum displacement of three story model under Tabas earthquake excitation

Model	Displacement (m)		
	Story 1	Story 2	Story 3
Numerical	4.18E-03	7.63E-03	1.36E-02
Identified	4.20E-03	7.53E-03	1.33E-02

CONCLUSIONS

The investigation on the responses of the identified model shows that the proposed method can be a strong tool for seismic evaluation of linear behavior of shear-type buildings. Although the identified stiffness matrices was not completely symmetric, but they can describe the linear dynamic behavior of their original systems and predict important dynamic responses in the seismic design as maximum story drifts and maximum base shears with an acceptable accuracy. Selection of minimum order of models is very important step that in practice is not so easy but has a crucial role in identifiying the correct and exact general matrices. The selected order can be confirmed by iteration with more responses or different initial order.

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